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Major E. H. HILLS, C.M.G., F.R.S., PRESIDENT, in the Chair.

William Herbert Henry Brookes, Principal Observer at the National Physical Laboratory, Ewelme, Claremont Road, Teddington, S.W.,

was balloted for and duly elected a Fellow of the Society.

Dr. Robert G. Aitken, Lick Observatory, Mount Hamilton, California, U.S.A.;
Professor Fritz Cohn, K. Astronomisches Rechen-Institut, Berlin; and
Professor Alfred Wolfer, Ph.D., Sternwarte, Zurich, Switzerland,

were balloted for and duly elected Associates of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

F. C. H. Carpenter, M.A., B.Sc., University Observatory, Durham (proposed by E. H. Hills);
Douglas Horsefield, St. Silas' Vicarage, Bristol (proposed by Rev. R. W. B. Moore);
Harold Spencer Jones, Jesus College, Cambridge (proposed by F. W. Dyson); and
Herbert George Robins, F.G.S., F.R.G.S., F.R.M.S., F.R.Met. Soc., Bulawayo, Southern Rhodesia (proposed by James Simms).

Seventy-seven presents were announced as having been received since the last Ordinary meeting, including, amongst others:—

A series of photographs of Comet 1911 *c* (Brooks), and photographs of Saturn, presented by the Yerkes Observatory. S. Newcomb: Researches on the Motion of the Moon, Part II. (American Ephemeris Papers), presented by the American Ephemeris Office. Astrographic Chart: 14 Charts, presented by the Specola Vaticana, Rome.

On some Spectrographic Measures of the Solar Rotation, made at the Kodaikánal Observatory. By J. Evershed and T. Royds, D.Sc.

During the dry seasons of 1911 and 1912 advantage was taken of days of exceptional clearness, when the sky appeared quite free from diffusive cirrus cloud, to obtain high dispersion spectra of the east and west limbs of the Sun and of the centre of the disc, for the purpose of studying the displacements towards the red of certain lines of iron and nickel at the Sun's limb.

The method of successive exposures was adopted, a plate being given a short exposure to the centre of disc spectrum, then a long exposure to the limb spectrum, and finally a second short exposure to the centre spectrum. The solar image was moved on the slit plate to the required position for each exposure, and Rowland's device of a movable slot immediately in front of the photographic plate was used to prevent overlapping of the limb and centre spectra. In this way two spectra representing the centre of the disc were obtained on either side of a central strip representing the limb.

Two series of exposures were made on each plate, one giving the spectrum of a point near the east limb, and the other a point nearly opposite on the west limb. Owing to the general displacement of the lines in the spectrum of all parts of the limb to the red as compared with the spectrum of the centre, the rotation displacements at the west limb are in general apparently greater than those at the east. If the spectra represent points at opposite ends of a solar diameter, and therefore in the same latitude, it may be assumed that half the difference of shifts (west-east) represents the limb displacement, and half the sum of the shifts the rotation displacement.

In the series of plates obtained it was intended to measure first the displacements in a few of the best-defined lines of each plate and get out the rotation velocity, merely as a check on the reliability of a plate, before proceeding to determine as accurately as possible the very minute limb displacements; for it was found that in a few cases, owing perhaps to unequal heating of the jaws of the slit, or to temperature changes in the grating during the

successive exposures, the values of the rotation speed, after the usual corrections, differed considerably from the accepted values.

In most cases, however, fairly consistent values were obtained, and if a plate gave a result within about 10 per cent. of Adams' values, it was considered suitable for estimating the limb shifts. This rather wide allowance was made because it was found that all the plates gave consistently smaller values than Adams of the order of 5 per cent.

At first this was considered to be due to some instrumental cause, and the spectrograph was improved in various ways in the direction of increasing the stability of the different parts. After this some high-dispersion spectra were obtained and measured of the a group of lines in the red, including the lines used by Dunér and Halm in their measures of the solar rotation. In these spectra the stability of the apparatus is proved by the telluric lines, which remain unshifted in the successive exposures. In the measures, however, the solar lines were referred to the telluric lines instead of measuring the direct displacements, so that there could be no question of a temperature effect.

The results from these plates (years 1911 and 1912) again showed smaller values of the rotation than those obtained by Adams in 1908.

The plates were not, however, as consistent amongst themselves as could be wished, the values ranging from 90 to 99 per cent. of Adams' values, and doubts still remained as to the possibility of errors arising from diffusion at the surfaces of the mirrors, there being four reflexions from silvered surfaces before the light reached the slit. The effect of these might be supposed to be the same as that of a diffusive sky in causing light from the whole disc of the Sun to enter the slit together with light from the limb.

The hypothesis that the Sun had actually slowed down in rotation speed, or that strong easterly winds prevailed in the region of the reversing layer, appeared to be less probable than that some error affected our results.

In the autumn of 1912 it was decided to make preparations for a new campaign in the approaching dry season, and to arrange for *simultaneous* exposures for east limb, west limb, and centre of disc. An 8-inch objective acquired from the Poona Observatory was substituted for the concave mirror previously used to form the solar image, and a large reflexion prism directs the light on to the slit, with a concave lens interposed to enlarge the image to 60 mm. diameter.

The diffusion of light with this arrangement is noticeably less than with the concave mirror and convex enlarging mirror previously used, and is probably less than is produced by the two cœlostat mirrors and concave mirror of the Snow telescope at Mount Wilson.

A special piece of apparatus was constructed for reflecting the light from two opposite points on the Sun's limb and from the exact centre of the disc on to the spectrograph slit, so that the

spectra from the three sources may be photographed simultaneously, special adjustments being provided for securing perfectly equal and uniform illumination of the grating from the different sources.

The main purpose in view was the measurement of the limb displacements, but it was felt that reliable values of the solar rotation ought to result also from the measures.

It was at this stage that we learned of the interesting results obtained by Dr. Hubrecht at the Cambridge Observatory. His results agree with ours in two ways, first in the lower equatorial speed found as compared with the usual values,* and secondly in the unexpectedly large variations between plates obtained at different dates.†

In view of the possibility of a real change of velocity between times of maximum and minimum sun-spots, and of real variations due perhaps to easterly or westerly currents in the reversing layer, it was resolved to take a series of plates during the early months of 1913, with the special object of measuring the equatorial speed with the greatest attainable accuracy, and it is the purpose of this communication to give a preliminary statement of the results obtained, and to draw attention to the apparent inconstancy of the rotation velocity when determined spectrographically.

A more complete statement of our results and a full description of the spectrograph employed must be deferred until all the observations for the year have been worked out, when they will be published in an Observatory Bulletin.

Observations in 1913.

Three series of spectra have been obtained covering the period January–April 1913.

Series I. were obtained with a special reflecting device, placed in front of the slit of the spectrograph, to enable simultaneous exposures to be made on the electric arc and the Sun.

The large concave mirror was used to form the image, which was enlarged to 120 mm. diameter by means of a convex mirror. The arc lines appear on either side of a narrow strip of spectrum representing a point on the Sun about $\frac{1}{24}$ of the radius inside the limb. Two spectra are photographed on each plate, one representing the east and the other the west limbs at latitudes not exceeding 8 degrees from the equator.

Series II. were obtained with the 8-inch object-glass to form the image and the reflecting device for simultaneous exposures on east limb, centre of disc, and west limb. The solar image is enlarged to 60 mm. diameter on the slit plate, and the limb spectra represent points usually $\frac{1}{30}$ of the radius inside the limb and at latitudes from 0° to 15°. The arc spectrum of iron is impressed on either side of the solar spectra.

* Observatory, December 1912, p. 421.
† *Monthly Notices*, vol. lxxiii. p. 15.

Series III. are similar in every respect to Series II., excepting that there is no central strip of spectrum representing the centre of the disc, the two limb spectra being contiguous.

In Series II. and III. the arc spectra are not photographed simultaneously with the solar spectra; they are used for the purpose of determining the correction to be applied for inclination of the micrometer thread to the spectrum lines.

As micrometer makers do not supply a ready means for rotating the thread, its adjustment exactly parallel with the spectrum lines is practically impossible, and serious errors may result if the exact inclination is not carefully measured and allowed for. The Fe arc lines afford an excellent means for doing this.

In Series I. the spectra on either side of the central strip being similar, no corrections for inclination of the wire are needed, settings being made on the arc line at equal distance above and below the solar spectrum.

It is evident that in Series I., in which each limb is separately compared with a terrestrial spectrum, the probable error of a velocity determination will in general be larger than in a measurement made when the opposite limbs are directly confronted, and the displacement is the result of twice the rotation speed. We considered, however, that it was of great importance to secure this series of plates and compare the results with Series II., taken on the same days but under somewhat different instrumental conditions. The method is analogous to that adopted by Dunér and others, who used the lines of terrestrial oxygen as intermediaries or standards of reference, but it has the advantage that the much sharper solar lines in the violet region can be used, and a higher order of spectrum utilised than is convenient for the red end.

It was hoped that by a comparison of the two methods adopted systematic errors affecting either might be detected, and it might also be possible to determine whether the large variations of velocity at different dates, previously found by the alternate method of exposure, were real or due to systematic errors.

The spectra were all taken with a Rowland grating of $3\frac{1}{4}$-inch ruling and 14438 lines to the inch, the 3rd order being used throughout. The scale of the plates varies considerably in different regions; 1 mm. is equal to 0·82 Å near K, and 0·55 Å near D.

Each plate includes a length of spectrum equal to about 100 Å.U. As it was desired to utilise Series II. for determining limb displacements over a great range of spectrum, it was found convenient to photograph the entire region from λ 3900 to λ 4800 in all three series, and after this had been done a number of plates of Series II. and III. were taken in the region λ 5300 to λ 5600. It was hoped that the different lines might give more consistent values of the rotation speed in this less refrangible region, since the linear dispersion and also $\Delta\lambda$ increase towards the larger wave-lengths. It was found, however, that the probable error of a velocity determination is about the same in the green region as in the violet near "K," notwithstanding the smaller dis-

placements in the latter region. This may be accounted for by the extremely good definition and narrowness of the lines in the violet.

The plates were measured and reduced by four observers, using two photo-micrometers by Hilger. Some of the plates of the green region were measured very successfully by sliding a positive copy on the negative, by means of a special piece of apparatus which can be attached to one of the micrometers. By this method, a full description of which will be published shortly, the intervals measured are double those between the displaced lines, or, for Series II. and III., four times the displacement due to the rotation speed of the Sun.

In the reduction of the measures a dispersion curve has been drawn for each plate and separate dispersion factors determined for each individual line. This is necessary, because the spectra are far from being normal and the change in 100 units is considerable.

In Table I. the results are given for the plates of Series I. and II., and in Table II. those for Series III. and II. In columns 4 and 5 of these tables v represents the observed velocity in kilometers per second corrected to the limb and for inclination of the Sun's axis to the line of sight.

TABLE I.

Deduced Sidereal Velocity at Equator (V) *for Series I. and II.*

Date. 1913.	Region.	ϕ.	Series.	v.	V.	Measurer.
		°				
Jan. 13	4760 – 4660	7·5	I	1·804	1·973	R
13	„	4·1	I	1·812	1·964	R
16	„	3·6	I	1·876	2·026	R
21	„	3·0	I	1·843	1·991	R
21	„	2·0	I	1·800	1·945	R
22	„	0·1	I	1·882	2·024	R
28	4640 – 4530	0·6	I	1·762	1·904	R
30	„	0·4	I	1·848	1·990	R
31	„	2·6	I	1·952	2·098	R, E, N, and F
Feb. 4	„	2·6	I	1·799	1·945	R
4	„	2·2	II	1·858	2·003	E
5	4530 – 4420	6·5	I	1·860	2·022	R
5	„	1·7	II	1·773	1·917	R
6	4410 – 4310	4·7	I	1·810	1·963	R
6	„	0·5	II	1·818	1·960	R
6	4522 – 4427	5·4	II	1·800	1·956	F
7	4410 – 4310	2·0	I	1·873	2·017	R
7	„	8·9	II	1·754	1·932	F
8	4290 – 4190	4·6	I	1·758	1·910	R
8	„	0·6	II	1·794	1·936	R
10	4200 – 4070	2·1	I	1·746	1·890	R
10	„	3·2	II	1·822	1·969	R
13	4080 – 3970	5·2	I	1·710	1·864	R
13	„	0·6	II	1·764	1·906	R
13	4200 – 4070	5·6	II	1·855	2·011	R

TABLE II.

Deduced Sidereal Velocity at Equator (V) *for Series III. and II.*

Date. 1913.	Region.	ϕ.	Series.	v.	V.	Measurer.
Mar. 11	3906 – 3971	10°·6	III	1·767	1·958	F
14	3990 – 4107	6·3	III	1·795	1·953	E, F
14	3998 – 4009	0·2	III	1·867	2·007	F
15	4118 – 4232	0·0	III	1·819	1·959	E, F
15	4232 – 4294	6·7	III	1·759	1·919	F
19	4376 – 4466	2·5	III	1·826	1·969	F
19	,,	0·1	III	1·835	1·975	F
19	4494 – 4590	13·0	III	1·709	1·924	F
20	,,	0·0	III	1·833	1·972	F
20	4603 – 4700	5·1	III	1·757	1·908	F
21	3906 – 3971	5·3	III	1·842	1·995	F
22	,,	1·4	III	1·863	2·003	F
26	,,	3·0	III	1·818	1·961	F
27	,,	14·4	III	1·736	1·968	F
28	5500 – 5600	0·1	III	1·730	1·868	E, F
Apr. 1	5346 – 5410	6·8	III	1·722	1·882	E
4	5569 – 5624	10·3	III	1·728	1·915	E, F
5	5562 – 5624	3·5	III	1·782	1·926	E
6	5569 – 5624	1·4	II	1·807	1·945	E, F
10	5543 – 5615	5·2	II	1·750	1·901	F
12	,,	15·6	II	1·591	1·838	E
12	,,	0·9	II	1·759	1·896	F

In the column V these values have been converted to sidereal velocities with the aid of Dunér's tables, and reduced to the equator on the assumption that the formula $(1{\cdot}507 + {\cdot}546 \cos^2 \phi) \times \cos \phi$ derived by Adams from his observations in 1908 represents the change of velocity with latitude with sufficient accuracy.

The last column in the tables gives the measurer's initial, E, F, N, R, designating Evershed, Miss Feline, Nagaraja, and Royds.

No large systematic differences between the observers have been found, and in five plates of Series II. and III. measured in duplicate by E and F, four agree within 0·005 km./sec. and one gives a difference of 0·031 km./sec. All the plates have been measured in both directions, viz. with the end towards the red to the right hand, and to the left on the micrometer.

As regards the lines used, in Series I. we were limited to the lines of Fe and Ni produced in the arc. In Series II. and III. the best-defined lines were selected, and include the following elements: Fe, Ni, Ti, Sc, Cr, Ca, V, Mg, Zr. We have not so far detected any systematic differences between different elements, and the variations

from line to line seem largely due to accidental errors of measurement, although in some cases we have strong reasons for suspecting errors peculiar to certain plates, due possibly to irregular shrinking of the film in drying.

The agreement between the plates of Series I. and II. obtained during February is not very good, although the mean results agree within 0·01 km./sec. It may be remarked, however, that there is always a difference of many degrees in the heliographic positions of the points photographed, more difference in fact than appears from the latitude column in Table I., because the points photographed on the same day by the two methods will usually be on opposite sides of the solar equator. It is conceivable, therefore, that a local current in the reversing layer may affect one plate and not the other, obtained on the same day but perhaps half an hour later, when the image will have rotated through several degrees.

We believe that there can be no serious errors in the latitudes determined for each plate. These are computed in the same way as is done daily for the prominence plates photographed with the spectroheliograph, and these are checked by visual observations with an equatorial telescope and position circle. The spectroheliograph is fed by the same siderostat which supplies light to the spectrograph, and the hour angle of the Sun at the time a photograph is obtained affords the datum for computing the position angle of the vertical point on the solar image. This is done with the aid of convenient tables giving the position angles corresponding with different hour angles and dates. The tables have been prepared from the formulæ of Cornu.*

Probable Errors.—The probable error of a plate for Series I. is approximately ±0·03 km./sec., where about seven lines are measured. For Series II. and III., in which ten to fifteen lines were measured on each plate, it is of the order of ±0·015 km./sec. More accordant values for the different lines are obtained by the method of sliding a positive copy on a negative, and the average p.e. of six such measures is ±0·009 km./sec., the best plate giving ±0·0045 km./sec. for twelve lines, and the worst (underexposed) ±0·015 km./sec. for eleven lines.

The probable error for a single line for Series I. is about ±0·08 km./sec.; for Series II. and III. the average of nine determinations is ±·052 km./sec., and the average of six determinations by the positive on negative method is ±·032 km./sec. The discordance between the different plates is of about the same order as that between the different lines on a plate for Series II. and III.

Results for the Different Months.—The statement in the last paragraph shows that the variations from plate to plate are not due to errors of measurement. As we can discover no instrumental cause, we conclude provisionally that real variations occur.

For convenience of comparison with results which may have been obtained at some of the co-operating observatories of the Solar Union, we group the results for the four months January to

* *Astrophysical Journal*, xi. 148.

April 1913 as follows, giving probable errors derived from the accordance of the different plates in a group.

Mean of	9	plates,	Jan. 13 to 31,	Series I.	V = 1·990 km/sec.	± ·0124
,,	7	,,	Feb. 4 to 13,	,, I.	V = 1·944 ,,	± ·0155
,,	9	,,	,, 4 to 13,	,, II.	V = 1·953 ,,	± ·0077
,,	15	,,	Mar. 13 to 28,	,, III.	V = 1·956 ,,	± ·0065
,,	7	,,	Apr. 1 to 12,	,, II. and III.	V = 1·900 ,,	± ·0087

It may be noted that all these values are considerably lower than Adams' results for 1908 (V = 2·053 km./sec.) :—

Series I.	January	being 3 per cent. lower.
,, I. and II. . . .	February	,, 5 ,, ,,
,, III.	March	,, 5 ,, ,,
,, II. and III. . .	April	,, 7½ ,, ,,

There seems strong evidence here of a progressive change, which, however, cannot be followed up effectively at this Observatory, owing to diffusive skies which are commonly prevalent after April and continue with very few days of sufficient clearness until December.

The large reduction of velocity in April coincides with a change from the violet to the green region of the spectrum. While admitting that we should have had greater confidence in the April results had we continued photographing in the violet, we can yet discover no reason why the less refrangible region should give systematically low values. The plates are excellent in definition and contrast, and are not difficult to measure. On April 4, without moving the grating, a plate was obtained of the fourth order violet overlapping the third order green. This was measured, and eleven lines in the region 4182–4126 gave the value V = 1·904 km./sec. ± ·0075, which is in agreement with the average results obtained from the green region.

Conclusions.—The possibility that large proper motions in the reversing layer may affect rotation measures has been discussed by Adams, who found differences of the order 0·13 km./sec. at the equator in the neighbourhood of sun-spot disturbances.*

Our results, so far as they go, would appear to indicate that variations amounting to 2 or 3 per cent. are of frequent occurrence at a time when there are very few spot disturbances. Larger progressive changes are also indicated, so that it is not possible to derive reliable values of the equatorial velocity from observations extending over a few months.

A possible connection between rotation speed and sun-spot disturbances may be worth mentioning. A small sun-spot was on the east limb at latitude + 15° on January 14, another on the west limb at lat. − 14° on January 16, and a third on the east limb at

* *An Investigation of the Rotation Period of the Sun*, p. 122. Mount Wilson Solar Observatory Papers, vol. i. part 1.

lat. – 11° on January 22. No more low latitude spots occurred until April 5, and this one disappeared before reaching the limb. The rotation results give high values for January, and the highest of all on January 16 and 22, when spots were very near the limb and not far from the region photographed.

It is suggested here that there may be a relation between higher rotation speeds and the development of spots; and that times of maximum spot development correspond with high values of the rotation, or rather, it should be said, large proper motion of the reversing layer in the direction of the Sun's rotation.

The interesting results of Mr. Hubrecht's latitude observations made in June 1911 seem favourable to this suggestion. By his method of observation he was able to discriminate between the two hemispheres of the Sun, and he obtains a larger rotation velocity in the southern hemisphere than in the northern.* During 1911 the southern hemisphere was far more active in spot production than the northern.

Kodaikánal:
1913 *May* 21.

On the Origin of Solar Electricity.

By J. A. Harker, D.Sc., F.R.S.

(*Communicated by the Astronomer Royal.*)

More than twenty years ago, in his presidential address to the Physics Section of the British Association at Edinbugh, when discussing solar phenomena, Professor Schuster † said:

"May not the periodicity of sun-spots and the connexion between two such dissimilar phenomena as spots on the sun and magnetic disturbances on the earth be due to a periodically recurring increase in the electric conductivity of the parts of space surrounding the sun?"

Professor Hale, ‡ in his Royal Institution lecture in 1909 on "Solar Vortices and Magnetic Fields," described the methods by which he had detected within the vortices, shown by the spectroheliograph to be present in sun-spots, magnetic fields of considerable strength. In discussing the possible origin of these fields, Hale says:

"We know from Rowland's experiment that the rapid revolution of an electrically charged body will produce a magnetic field. Thus if a sufficient number of electrically charged particles were set into rapid revolution by the solar vortices, a magnetic field should result. What warrant have we for assuming the existence of charged particles in the sun?"

* *Monthly Notices*, lxxiii. p. 25.
† Schuster, *Brit. Assoc. Report*, 1892, p. 635.
‡ Hale, *Proc. Roy. Inst.*, xix. p. 621.

Some experiments made at the National Physical Laboratory during the past two years by Dr. G. W. C. Kaye* and the writer on the emission of electricity from hot bodies seem to have a distinct bearing on these problems. The application to cosmical phenomena of some deductions from these and a few hitherto unpublished further experiments forms the subject of the present paper.

It may be well briefly to recapitulate here the results attained. The first experiments dealt with the electric conductivity of the space inside a carbon-tube resistance furnace heated by alternating current. A pair of small insulated exploring electrodes of carbon projected into the furnace at opposite ends, and were connected with a battery of a few volts and a suitable current-measurer. It was found that, as the furnace was gradually heated from room-temperature, the resistance of the interior space remained practically infinite until a temperature of about 1400° C. was reached, when the resistance commenced to fall rapidly. The curve showing the current between the two exploring electrodes was exponential in character.

It was further found that, on removing the battery, and therefore in complete absence of all applied potential, a transient electric current could be obtained by keeping one of the electrodes in a fixed position in the furnace, and heating or cooling the other by moving it in or out of the hot region. The direction of the current indicated a negative † emission from the hotter to the colder surface across the gap.‡

Ionisation currents up to about 3 amperes at over 3 volts were thus obtained from a quite small apparatus, having a working electrode-area of only a few square centimetres. If both electrodes are kept stationary in the furnace, and one is maintained permanently hot and the other, by means of water-cooling, kept at a much lower temperature, continuous currents of the order of an ampere can be obtained without the application of any potential.

In a later form of the experiment a thin rod of carbon about 15 cm. in length was heated to a very high temperature by low-voltage alternating current; an ionisation current of over $3\frac{1}{2}$

* Harker and Kaye, "On the Emission of Electricity from Carbon at High Temperatures," *Proc. Roy. Soc.*, vol. lxxxvi., A, 1912, 379; "On the Electric Emissivity and Disintegration of Hot Metals," *Proc. Roy. Soc.*, June 19, 1913.

† Positive emissions were obtained with new carbon at low temperatures on first heating, and were probably associated with the expulsion of the first traces of the contained impurities.

‡ The hot fixed electrode, being in thermal equilibrium with its surroundings, takes no part in the generation of these currents. The latter may be considered as entirely due to the alternating temperature differences between the surface of the movable electrode and the hotter and colder parts of the furnace-wall immediately surrounding it. It has been conclusively established that the ionisation currents are *not* due to thermo-electricity, or to any rectifying action of the carbon on the alternating currents used for heating the furnace. Many of the effects have been obtained in experiments where the heat-supply has been furnished from non-electrical sources.

amperes was emitted by the surface of the rod to a comparatively cool insulated carbon tube surrounding it, the current-density in this case exceeding 0·1 ampere per sq. cm. at atmospheric pressure.

The current appears to be associated with the emission from the hot carbon surface of negatively charged particles. As all the experiments were made at atmospheric pressure, it would seem that these particles must consist almost entirely of sputtered *matter*, and that "corpuscles," whose unencumbered range at such pressures would be extremely small, do not here play the great part in the phenomena they do in high vacua. The evidence to hand in support of this view is discussed in the two memoirs referred to.

In view of the interest of these results obtained with carbon, preliminary experiments on the volatilisation and electric emissivity of a number of metals were made, mostly in nitrogen, at pressures from a few millimetres upwards. The metals were heated by alternating current, and no applied potential was employed. It was found that positive electricity was emitted at temperatures from 1000° to 1400° C. With those metals which melt within this range, a sudden and marked increase in the positive current occurred at the liquefying-point.

At higher temperatures negative electricity predominates, and its quantity increases rapidly with the temperature. With strips of pure iridium, melting between 2300° and 2400° C., which was the metal found to give by far the largest emission, the negative current attained nearly $\frac{1}{10}$th of an ampere at the melting-point, the radiating surface being approximately 4 sq. cm. With iron transformer-plate melting at about 1500° C., a large emissivity was often obtained, but with this metal it was found impossible to repeat any given set of conditions sufficiently accurately to obtain even approximately identical results in successive experiments. The negative currents appear to be always associated with considerable sputtering of the metal, and are probably a consequence of some reaction between the metallic surface and the surrounding gas. With carbon the nature, and to some extent also the pressure, of the gaseous atmosphere surrounding the radiating material exerted an influence on the emissivity at the higher temperatures, and with the metals the temperature at which the positive emission changed sign seemed to be greatly influenced by the presence or absence of minute quantities of impurity in the nitrogen atmosphere surrounding the specimen.

From these experiments, and from the known facts regarding the emissions from other materials such as refractory oxides, it would appear that *most substances capable of withstanding a high temperature emit electricity when strongly heated, and that their emissivity when once established increases rapidly with rise of temperature.*

It was obvious from all the experiments that the electric conductivity inside a hot carbon-tube furnace, as evidenced by the ionisation currents produced, was very large, and it is of interest to compare the gaseous conductivity thus manifested with other examples.

A hard Röntgen-ray tube has an apparent resistance of megohms. A quartz mercury-lamp of the usual type has a resistance when running of the order of 50 ohms. In many of these experiments, even with quite small furnaces, the gaseous resistance was measured to be less than 1 ohm at temperatures above 2000° C.; while in one case in a large furnace, where the temperature was pushed to over 3000° C., the apparent resistance fell to microhms, or at any rate to something comparable with that of the graphite rods and of the copper cables forming the rest of the electric circuit. This would mean that *the conductivity of the ionised space was of the same order as if it were filled with mercury or a liquid electrolyte.*

In none of the emission experiments either on carbon or metals up to their melting-point was there any evidence of discontinuity in the law of rapid increase in emissivity with temperature, which was found to hold for the solid substance. Some of the metals, however, showed on melting a marked *increase* in the emission at the moment of fusion.

A word or two may be said here as to the probable velocity and size of the particles which apparently act as carriers in these experiments. In many instances no electric potential was applied, and it seems, therefore, that the particles owe their velocity of emission solely to the temperature. For carbon at 3000° C., according to J. J. Thomson's theory, this velocity should be about 3×10^7 cm. per second, or about a thousandth of that of light, a speed which, compared with that of some corpuscles, is very slow. But, unlike corpuscles, the particles can travel considerable distances in gases even at atmospheric pressure.

If, as we are inclined to believe, it should ultimately prove to be the case that the ionisation currents are carried chiefly by these particles, we have then a means of deducing fairly readily the relation of their charge to their mass (e/m).

An experiment to this end on iridium in nitrogen at 20 mm. pressure, the strip being at a temperature of 1360° C. (black-body), gave the result that the average number of iridium atoms contained in the particle carrying a single negative charge was 1200. In this case, therefore, the mass of the carrier is roughly

$$5 \times 10^8 \text{ times}$$

greater than that of a corpuscle, whose mass is $\frac{1}{1800}$th of that of a hydrogen atom.

Turning now to the phenomena presented in the sun, it would seem in the light of these facts impossible to avoid the conclusion that *the solar atmosphere is a highly conducting medium.* The most modern determinations of solar temperatures all give values between 5600° and 6000° Abs.,* estimates much lower than those

* For example, Abbot, *Measurements on Mount Wilson and Mount Whitney*, 5840° Abs. Calculated from an assumed value of Stefan's constant. Also Harker and Blackie, *National Physical Laboratory Report*, 1908. 5610° Abs. By total radiation pyrometer directly standardised up to arc temperature, assuming atmospheric absorption 29 per cent.

formerly accepted. From spectroscopic evidence it has been established beyond dispute that iron forms one of the principal constituents of the sun, and carbon a not unimportant one both of the sun and of many stars. For present purposes we will limit our considerations to a discussion of the probable influence on the solar atmosphere of the presence of these two elements.

Iron at its melting-point gave negative currents up to about 15 micro-amperes per sq. cm. in nitrogen at 9 mm. pressure; a rise of temperature of only 60° C. serving to double the current-density over this part of the range. Even assuming no further increase of emissivity with rise of temperature beyond the melting-point, the current emitted per square kilometre of the area of the photosphere would even on this basis attain 150,000 amperes.

Considering next the case of carbon, if we assume the validity for extrapolation purposes of any such law of increase of conductivity of the space inside a carbon furnace, as that deducible from the data plotted in fig. 4 of the paper alluded to,* we find that at a temperature far short of that of the sun the conductivity becomes greater than that of any known substance.

Suppose however that, to be on the safe side, we assume in the sun no further increase in emissivity above the figure of ·1 ampere per sq. cm., actually measured at a temperature approximately 3000° C., this would be more than ample to account for the enormous currents, which recent considerations have rendered it probable are circulating in the solar atmosphere.

Hale has shown that it is highly probable that intense magnetic fields of from 2000 to 5000 C.G.S. units are found in sun-spots, and that there is also a general magnetic field of much less intensity existing over the whole body of the sun. Assuming that the circuit producing the field in a spot consists of a single turn, calculation shows that currents of the order of ten million million amperes are required.†

But at the rate of ·1 ampere per sq. cm., which is 1000 million amperes per sq. kilo., an area of only 100 × 100 kilometres would alone be required to generate this current, assuming carbon as its source.

As to the mechanism by which the currents *are actually generated* in the solar atmosphere, our present knowledge is very meagre, particularly in view of the great uncertainties as to the physical state of the elements in the sun at the prevailing pressures, also as regards temperature distribution over the surface of the photosphere, and radially outwards through the solar atmosphere. Of the several alternative processes by which the enormous local currents circulating round a sun-spot *might* be generated, that favoured by Hale, the revolution of large numbers of the charged particles by the solar vortices, seems least open to objection.

One obvious direction for further research, before the data

* Harker and Kaye, *loc. cit. Proc. Roy. Soc.*, vol. lxxxvi., A, 1912, p. 385.

† The Astronomer Royal and Mr. Maunder kindly supplied the necessary dimensional data for the spots to enable this computation to be made.

here discussed can be properly applied to the elaboration of an electrical solar theory, is the continuation of the experiments on metals beyond their melting- and boiling-points. Indeed, a few preliminary experiments in this direction have already been made. One obstacle at present in the way of their successful completion is to find a material suitable for the construction of the necessary vessels, which is at the same time sufficiently inert and refractory, and which does not itself emit electricity over the temperature range to be studied.

Some indirect evidence on this question of the behaviour of vapours is, however, to hand. It was found that in the experiments with carbon, when a new carbon tube of ordinary commercial quality was heated for the first time, the emission at temperatures in the neighbourhood of 2000° C. was always four or five times as great as in later heatings to the same temperature. It was always observed that, during the first heating in a current of inert gas, the impurities in the carbon, consisting mostly of iron and silicon, are distilled out of the body of the material and carried off in the stream of gas. It was ascertained that this comparatively volatile material all leaves the carbon surface carrying a charge.

In bringing the foregoing facts to the notice of astronomers and cosmical physicists, the writer is fully conscious of many criticisms which may be urged against the conceptions here outlined. But in view of the discoveries of Hale and others, and the consequent interest now being taken in solar phenomena, a presentation of the data even at this early stage in their investigation may not be considered premature.

It may ultimately prove to be the case that such evidence as is here presented, if further supported, may be considered by the authorities as sufficiently conclusive for presuming the existence in the solar atmosphere in very large numbers of electrified particles, by the movements of which enormous currents are generated. Very probably these particles have a mass and velocity widely different from those of corpuscles, the bodies hitherto considered almost exclusively in discussions as to solar currents and magnetism.* It may therefore, as a consequence, be necessary for cosmical physicists to revise some of their arguments as to the interrelations of terrestrial and solar magnetic phenomena.

In conclusion, I desire to express my thanks to the Astronomer Royal, to Professor Fowler, and to Dr. W. J. S. Lockyer for their help in references to data and literature.

Eskdalemuir Observatory:
1913 *May* 17.

* See, for example, Schuster, *Proc. Roy. Soc.*, 1911 (1), p. 44; and Birkeland, *The Norwegian Aurora Polaris Expedition*, vol. i.: *On the Cause of Magnetic Storms and the Origin of Terrestrial Magnetism*, pp. 151 *et seq.*

Anomalous Dispersion in Solar Phenomena. By D. Brunt, B.A., Isaac Newton Student in the University of Cambridge.

§ 1. Introductory.

The following investigation was undertaken in order to determine how far anomalous dispersion might be regarded as an important factor in accounting for solar phenomena. It was hoped that the measurements of anomalous dispersion made by Wood, Bevan, Ladenburg, and others, in conjunction with the three chief statical theories of the equilibrium of the solar atmosphere—viz. isothermal, adiabatic, and radiative equilibrium—might be applied to decide how far the conclusions drawn by Julius in his papers on anomalous dispersion in solar phenomena could be justified. But as these three theories yield a much less extensive atmosphere than is actually observed on the Sun, and so do not represent the observed phenomena even approximately, the discussion had to be conducted along lines which are independent of any theory of solar equilibrium and simply assume the existence of an extended atmosphere.

It was found that the measurements of anomalous dispersion in vapours and gases could not be used, as, in general, the density of the vapour employed, and the number of molecules per unit volume taking part in producing anomalous dispersion, were not given; nor were they capable of being deduced from the data, even within rough limits. Further, no data that would serve to give the density and density gradients in the solar atmosphere are at present forthcoming. Hence it is found necessary to base the discussion upon a consideration of the possible *arrangements* of the density gradients rather than upon their *magnitudes*, such arbitrary magnitudes of the density gradients being adopted as would suffice to produce the effects which are attributable to anomalous refraction.

In order to gain some idea of the densities which might be expected to exist at different heights above the photosphere, the three theories of solar equilibrium mentioned above are considered in some detail; and as prominences often extend to heights of 5′ above the photosphere, the density and density gradient at this height are evaluated on all three theories. The radius of the "critical sphere" is calculated and compared with the apparent radius of the photosphere. The magnitude of the effect produced by anomalous scattering is compared with the effect of true absorption. The different solar phenomena are then considered in detail. In the case of the prominences and chromosphere no criterion is found to decide between the anomalous refraction theory and the selective emission theory; but in the case of the Fraunhofer lines, spectroheliograph results, and sun-spots, it is shown that anomalous refraction cannot be an important factor in producing the observed phenomena.

For the sake of convenience in writing, light whose wave-length

is slightly greater than that of a true absorption line, is called R-light, while light whose wave-length is slightly less than that of a true absorption line is called V-light. Spectroheliograms taken with R-light or V-light are called R- or V-spectroheliograms respectively. The term "anomalous dispersion" is used for that property of a vapour in virtue of which the index of refraction of rays in the neighbourhood of an absorption line is subject to rapid variation with wave-length; while the term "anomalous refraction" has been used, in accordance with Julius's convention, to denote the refraction effect which enters in consequence of this variation in the index of refraction, when there is a gradient of density in the medium.

§ 2. The Equilibrium of the Solar Atmosphere.

The equations to be employed can be written in a simple form by the use of the following system of units and constants introduced by Schwarzschild.* A full account of the derivation of the equations will be found in Schwarzschild's paper.

$g = 27{\cdot}7$ = ratio of the gravitational attraction at the photosphere to the gravitational attraction at the Earth's surface.

M = mean molecular weight of the solar atmosphere. (For air M = 28·9.)

R = 0·106 = gas constant.

t = absolute temperature in degrees centigrade at height h above the photosphere.

Unit of h = height of homogeneous terrestrial atmosphere = 8 km.

ρ = density at height h above the photosphere.

Unit of ρ = density of the terrestrial atmosphere.

t_0 = temperature of photosphere (generally estimated at 6000° C.).

ρ_0 = density at the photosphere.

a = radius of photosphere = $8{\cdot}75 \times 10^4$ units.

k = ratio of specific heats = $\frac{5}{3}$ or $\frac{7}{5}$ for mon- or di-atomic gases.

κ = constant = 0·6.

τ = constant temperature = 5050° C. $\quad 2\tau^4 = (6000)^4$.

We shall proceed to calculate the density and density gradient at a height of 5′ above the photosphere, on each of the three hypotheses.

(*a*) *Isothermal Equilibrium.*

In this case the temperature t will be constant throughout. If the mass of the solar atmosphere be neglected in comparison with the mass contained within the photosphere

$$\rho = \rho_0 e^{-\frac{M \cdot ga}{Rt} \cdot \frac{h}{a+h}}.$$

* *Nachrichten k. Gesellschaft zu Göttingen*, 1906, p. 41.

At height 5′ above the photosphere $\frac{h}{a+h}=\frac{5}{21}$,

and $$\rho=\rho_0 e^{-5\cdot44\cdot 10^6\frac{M}{t}}=\rho_0 10^{-2\cdot36\cdot 10^6\frac{M}{t}},$$

$$\frac{d\rho}{dh}=-152\frac{M\rho}{t},$$

or the gradient of density per centimetre $=1\cdot8\times10^{-4}\frac{M\rho}{t}$.

When we change into C.G.S. units, ρ_0 cannot exceed the mean density of the Sun, or 1·38. We shall give ρ_0 the value unity in the above equation, and so obtain values of ρ expressed in gm. cm.$^{-3}$.

When $t=6000°$ C., $\rho=10^{-393M}$; and it is clear that the solar atmosphere cannot extend to a height of 5′ above the photosphere. The values of the density and density-gradient can only be increased by decreasing $\frac{M}{t}$, *i.e.* by increasing the temperature, and assigning a low value to M. The values of the density and density-gradient at a height of 5′ above the photosphere are given in Table I., for a range of values of t, and for $M=2$ and $M=30$.

TABLE I.

	Density (gm. cm.$^{-3}$).		Density-gradient (gm. cm.$^{-4}$).	
	M=2	M=30	M=2	M=30
6,000° C. Abs.	10^{-786}	10^{-11790}	10^{-793}	10^{-11796}
10,000	10^{-472}	10^{-7080}	10^{-480}	10^{-7087}
100,000	10^{-47}	10^{-708}	10^{-56}	10^{-716}
1,000,000	2.10^{-5}	10^{-71}	7.10^{-15}	10^{-80}
2,000,000	5.10^{-3}	10^{-35}	10^{-12}	10^{-43}
5,000,000	10^{-1}	10^{-14}	10^{-11}	10^{-23}

(*b*) *Adiabatic Equilibrium.*

Schwarzschild * gives the following relation for a solar atmosphere in adiabatic equilibrium :—

$$t_0-t=\frac{k-1}{k}\cdot\frac{Mg}{R}h.$$

The gravitational attraction is assumed to have the constant value g throughout the region considered. The outer limit of a sphere of gas in adiabatic equilibrium is determined by $p=\rho=t=0$; and so the above equation may be applied to determine the height to which the solar atmosphere extends above the photosphere. If the temperature t_0 of the photosphere be 6000° C., the height of an atmosphere of diatomic hydrogen would be 300 km., and the height of an atmosphere similarly constituted to the terrestrial atmosphere 22 km. In order that the solar atmosphere

* *Nachrichten k. Gesellschaft zu Göttingen*, 1906, p. 41.

should reach a height of 5′, the temperature of the photosphere would have to be 4,600,000° C. for an atmosphere of hydrogen, and 60,000,000° for an atmosphere of air, and in order to obtain a finite density at this height we should have to assign a still higher temperature to the photosphere.

(c) *Radiative Equilibrium.*

This theory is based on the assumption that each layer of the solar atmosphere takes up by absorption as much energy as it loses by radiation, its temperature remaining unchanged. Schwarzschild * derives the following relations between h, ρ, and t:—

$$-h = b\left[\frac{4t}{\tau} + \log_e\frac{t-\tau}{t+\tau} - 2\tan^{-1}\frac{t}{\tau} - 0{\cdot}616\right],$$

$$\rho = \frac{Mg}{R\kappa}\,\frac{t^4-\tau^4}{t\tau^4}.$$

τ is defined by the relation $2\tau^4 = (6000)^4$, and $\log b = \bar{1}{\cdot}8252010$. h is here measured from that layer of the solar atmosphere whose temperature is 6000° C., but this does not involve any assumption as to the actual temperature of the photosphere. In Table II. the values of h and ρ are given for different values of t. The tabulated values of ρ are expressed in C.G.S. units.

Table II.

h (in km.).	h (angular measure).	t	$t-\tau$	ρ gm. cm.$^{-3}$.
∞	∞	5,050° C.	0	0
210,000	5′	...	10^{-1630}	10^{-1636}
42,000	1′	...	10^{-326}	10^{-333}
3,500	5″	...	10^{-28}	10^{-34}
397	·5″	5,051	1	3.10^{-6}
275	·4″	5,060	...	3.10^{-5}
0	0	6,000	...	3.10^{-3}
−97	−·14″	7,570	...	$8{\cdot}8.10^{-3}$
−219	−·31″	10,100	...	2.10^{-2}
−4,085	−5·8″	101,000	...	3.10
−12,660	−17·5″	303,000	...	7.10^{2}
−210,000	−5′	5,050,000	...	3.10^{6}

From this table it is clear that the solar atmosphere would only extend to a height of a few seconds above the level where the temperature is 6000° C. It should be noted that the table may be used to calculate the temperature which must be assigned to the photosphere in order that the solar atmosphere should extend to a height of 5′. For the purpose of such an estimate the atmosphere may be regarded as ending at a level where the temperature is

* *Nachrichten k. Gesellschaft zu Göttingen*, 1906, p. 45.

6000°. It is seen that the temperature which must be assigned to the photosphere is about 5,000,000° C.

With the usually accepted values for the temperature of the photosphere, each of the three forms of equilibrium considered above would yield an apparently sharply bounded Sun, with an atmosphere extending to only a few seconds of arc above the photosphere. As the solar atmosphere is much more extensive than these theories would lead us to expect, it seems legitimate to conclude that in the solar atmosphere the effect of gravitation is very largely counteracted by other forces, such as light-pressure, or the effects of the emission of charged particles.

§ 3. The Radius of the Critical Sphere.

The units introduced in § 2 will be used in the following calculations. Further, let

e = distance of Earth from Sun.

γ = angular semi-diameter of the Sun = 1000″ approximately.

$e \sin \gamma$ = radius of photosphere = $8{\cdot}75 \times 10^4$ units of length.

r = distance of a point in the solar atmosphere from the centre of the Sun.

μ = index of refraction in the medium at that point.

$\mu = 1 + K\rho$, approximately, K being constant for a given wave-length.

a_1 = radius of the critical sphere.

μ_1, ρ_1 = index of refraction and density at the surface of the critical sphere.

Now the critical sphere is determined by the two equations *

$$\frac{d}{dr}(\mu r) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\mu r = e \sin \gamma = \mu_1 a_1 \quad . \quad . \quad . \quad (2)$$

We can now express μ as a function of r for each of the three forms of equilibrium, and can therefore proceed to calculate the radius a_1 for the three cases.

(*a*) *Isothermal Equilibrium.*

Let $r = a_1 + x$. Then the equation of § 2 (*a*) can be written in the form

$$\rho = \rho_1 e^{-\frac{Mga_1}{Rt}\frac{x}{a_1+x}}$$

$$\mu r = (a_1 + x)\left\{1 + K\rho_1 e^{-\frac{Mga_1}{Rt}\frac{x}{a_1+x}}\right\},$$

$$\frac{d}{dr}(\mu r) = \frac{d}{dx}(\mu r) = 0 \quad \text{at} \quad x = 0,$$

$$\therefore \quad 1 + K\rho_1 - K\frac{Mg}{Rt} a_1 \rho_1 = 0.$$

* Vide Wilczynski, *Astrophysical Journal*, i. 112.

We also have $e \sin \gamma = \mu_1 a_1 = a_1(1 + K\rho_1)$.

Eliminating $K\rho_1$ we find,

$$a_1^2 - e \sin \gamma a_1 + \frac{Rt}{Mg} e \sin \gamma = 0,$$

$$\therefore \quad a_1 = e \sin \gamma \left(1 - 4{\cdot}3\ 10^{-8} \frac{t}{M}\right),$$

or the difference between the angular radii of the photosphere and critical sphere $= 4{\cdot}3 \times 10^{-5} \frac{t}{M}$ seconds of arc.

This difference, expressed in seconds of arc, is tabulated below for a range of values of M and t.

TABLE III.

M	6000° C.	10,000° C.	100,000° C.	300,000° C.	1,000,000° C.
2	0″·13	0″·21	2″·1	6″·4	21″·
20	·013	·021	·21	·64	2·1
30	·009	·014	·14	·42	1·4

(*b*) *Adiabatic Equilibrium.*

Let $x + a_1 = r$.

Then the equations may be written in the form

$$\frac{\rho}{\rho_1} = \left(\frac{t}{t_1}\right)^{\frac{1}{k-1}}$$

$$t - t_1 = -\frac{k-1}{k} \frac{Mg}{R} \cdot x.$$

$$\frac{t}{t_1} = 1 - \frac{k-1}{k} \frac{Mg}{Rt_1} x.$$

$$\therefore \quad \frac{\rho}{\rho_1} = 1 - \frac{1}{k} \frac{Mg}{Rt_1} x, \text{ approximately}$$

$$\mu = 1 + K\rho = 1 + K\rho_1 \left(1 - \frac{1}{k} \frac{Mg}{Rt_1} x\right).$$

Now $$\frac{d}{dr}(\mu r) = \frac{d}{dx} \cdot \mu \cdot (a_1 + x) = 0 \text{ at } x = 0,$$

$$\therefore \quad 1 + K\rho_1 - \frac{KMg}{kRt_1} \rho_1 a_1 = 0.$$

Also $$(1 + K\rho_1) a_1 = e \sin \gamma.$$

Eliminating $K\rho_1$ and solving the resulting quadratic for a_1, we obtain the result $a_1 = e \sin \gamma \left(1 - \frac{kR}{g\, e \sin \gamma} \frac{t_1}{M}\right)$. From this it is clear that the difference in radii of the two spheres is small; hence it is legitimate to put $t_1 = t_0 =$ temperature of the photosphere. The differences in angular radii of the critical sphere and photosphere, expressed in seconds of arc, are tabulated in Table IV. for a

diatomic gas. For a monatomic gas the values would be increased in the ratio 25 : 21.

TABLE IV.

M	6000° C.	10,000° C.	100,000° C.	300,000° C.	1,000,000° C.
2	0".185	0".31	3".1	9".3	31"
20	".018	".03	".3	".9	3".1
30	".012	".02	".2	".6	2".0

(*c*) *Radiative Equilibrium.*

The density and temperature at height h above a given level are determined by the equations

$$\rho = \frac{Mg}{R\kappa}\,\frac{t^4-\tau^4}{t\tau^4}$$

$$-\frac{Mg}{R\tau}\,.\,h = \frac{4t}{\tau} + \log_e\frac{t-\tau}{t+\tau} - 2\tan^{-1}\frac{t}{\tau} + \text{constant}.$$

$$\frac{d}{dr}(\mu r) = \frac{d}{dh}(\mu r) = 1 + K\rho + rK\frac{d\rho}{dr} = 0 \text{ at critical sphere.}$$

$$\therefore \quad 1 + K\frac{Mg}{R\kappa}\,\frac{t_1^4-\tau^4}{t_1\tau^4} - Ka_1\,\frac{Mg}{R}\,\frac{t_1^4-\tau^4}{t_1\tau^4}\,.\,\frac{Mg}{R\kappa}\,\frac{3t_1^4+\tau^4}{t_1^2\tau^4} = 0.$$

Also $$\mu_1 a_1 = e\sin\gamma.$$

Eliminating a_1 from these two equations, and substituting for μ_1 in terms of t, we obtain the following equation involving only t_1

$$\left(1 + \frac{KMg}{R\kappa}\,\frac{t_1^4-\tau^4}{t_1\tau^4}\right)^2 = \frac{KM^2g^2}{R^2\kappa}\,e\sin\gamma\,\frac{(3t_1^4+\tau^4)(t_1^4-\tau^4)}{4t_1^2\tau^4}.$$

Let $t_1 - \tau = y$. Substituting the values of the constants in the last equation, and putting $M = 28{\cdot}9$ and $K = {\cdot}0003$, for an atmosphere of air, and omitting the terms in y^2, whose coefficients are very small, we find

$$1 + 3.10^{-6}y = {\cdot}8y \qquad y = 1{\cdot}25°.$$

Thus the temperature at the critical sphere is 5051·25° C. Table II. shows that the angular radius will differ from that of the sphere at whose surface the temperature is 6000° C. by less than ·5". It should be remembered that these calculations are based on the assumption that the constitution of the solar atmosphere is similar to that of the terrestrial atmosphere.

On each of the three theories of the equilibrium of the solar atmosphere we find that, with moderate assumptions for the temperature, and for the mean molecular weight of the atmosphere, the difference between the angular radii of the critical sphere and the apparent photosphere is a small fraction of a second of arc. Throughout the remainder of this paper it will be assumed that these two spheres are indistinguishable.

§ 4. Anomalous Scattering.

Rayleigh has shown that when a beam of light of wave-length λ, and intensity I_0 passes through a layer of gas of thickness z, in which there are N scattering molecules per unit volume, and in which the index of refraction is n for wave-length λ, the intensity of the beam is reduced to I, where $I = I_0 e^{-sz}$; if $s = \frac{32\pi^3(n-1)^2}{3N\lambda^4}$, s is called the coefficient of scattering.

Julius * assumes that this formula is perfectly general, and holds approximately even in the neighbourhood of an absorption line, where $n-1$ may be subject to very rapid changes. If this be true, then the rays which suffer anomalous dispersion also undergo a selective scattering effect. Julius gives this selective scattering effect the name "anomalous scattering."

Now the line of argument by which Rayleigh obtains the formula quoted above is not valid when there is resonance between the incident beam of light and the molecules of the medium, *i.e.* when the medium absorbs some of the light; and so the assumption of the truth of the formula in the neighbourhood of an absorption line is not justifiable. The effect of a resonating molecule on a beam of light, when part of the incident beam is scattered or diffracted by the molecules, another part absorbed, and the remainder transmitted unchanged, is beyond the range of mathematical investigation in the present state of our knowledge of the mechanism of radiation. Nor has the general problem of selective scattering ever been investigated experimentally. Lamb † has investigated mathematically the scattering effect of a molecule having one or more free periods, and having a very large specific inductive capacity. Lamb attributes absorption to this scattering effect. His results show that, with his assumptions, the width of an absorption line should be measured in 10,000ths of an A.U.

In the absence of any other working hypothesis, we are forced to assume, with Julius, that Rayleigh's formula holds in the neighbourhood of an absorption line. The joint effect of absorption and scattering is obtained by substituting $k+s$ for s in the formula for I quoted above, k being the coefficient of absorption. An idea of the relative importance of absorption and anomalous scattering can be obtained by evaluating $\frac{s}{k}$ in the neighbourhood of an absorption line. We shall evaluate this ratio for sodium, using Hallo and Geiger's magneto-optic observations, and for hydrogen, using Ladenburg's observations.

Regarding absorption and dispersion as resonance phenomena, Voigt ‡ developed the theory from the standpoint of the electron theory. Let ν_0 be the frequency of the free vibration of an

* *Le Radium*, vii., Oct. 1910; *Proc. Roy. Acad. Amst.*, xiii. 881–897, 1263–1273.

† *Transactions Camb. Phil. Soc.*, xviii. 348.

‡ *Magneto- und Elektrooptik*, p. 103 *et seq.*

electron, N′ the number per unit volume of electrons vibrating with this frequency, n the index of refraction, and κ the index of extinction for rays of frequency ν in the medium considered; and further, let $\mu = \nu - \nu_0$, $\rho = 4\pi N' \frac{e^2}{m}$, where m is the mass of an electron, and e its charge, and ν' is a quantity which depends on the frictional term in the motion of the electron. Voigt derives the following equations:—

$$n = n_0 - \frac{\rho\mu}{n_0\nu_0(4\mu^2 + \nu'^2)} .$$

$$n\kappa = \frac{\rho\nu'}{2n_0\nu_0(4\mu^2 + \nu'^2)} .$$

In Hallo's * experiments on the magnetic rotation in a sodium flame, the values of the parameters entering into the above equations were:—

$\nu_0 = 3{\cdot}2 \,.\, 10^{15}$. $\rho = 7{\cdot}6 \,.\, 10^{23}$. $\nu' = 1{\cdot}12 \,.\, 10^{11}$. $N' = 2{\cdot}5 \,.\, 10^{14}$.
$N = 5 \,.\, 10^{16}$. = number of molecules per unit volume.

Geiger's † observations of the magnetic rotation in a sodium flame yield the following values:—

$\nu_0 = 3{\cdot}2 \,.\, 10^{15}$. $\rho = 2{\cdot}2 \,.\, 10^{23}$. $\nu' = 2 \,.\, 10^{11}$.
$N' = 7 \,.\, 10^{13}$. $N = 7 \,.\, 10^{15}$.

Ladenburg and Loria's ‡ experiments on the absorption and dispersion of luminous hydrogen yield

$\nu_0 = 2{\cdot}9 \,.\, 10^{15}$. $\rho = 1{\cdot}2 \,.\, 10^{22}$. $\nu' = 1{\cdot}1 \,.\, 10^{12}$.
$N' = 4 \,.\, 10^{12}$. $N = 2 \,.\, 10^{17}$.

The quantity n_0 which enters into the equations given above differs only slightly from unity, and a fair approximation can be obtained by equating it to unity throughout. We then find

$$\frac{s}{k} = \frac{32\pi^3}{3N\lambda^4} \cdot \frac{(n-1)^2 \,.\, \lambda}{2\pi n\kappa} = \frac{32\pi^2 \,.\, \rho}{3\lambda^3 \,.\, N\nu_0\nu'} \cdot \frac{\mu^2}{4\mu^2 + \nu'^2},$$

$$= \frac{4\pi\rho}{3N\lambda^2\nu' c} \left\{ 1 - \frac{\nu'^2}{4\mu^2 + \nu'^2} \right\}.$$

The ratio of the two coefficients is thus never greater than $\frac{4\pi\rho}{3N\lambda^2\nu' c}$. Substituting in this expression the values of Hallo, Geiger, and Ladenburg and Loria, we obtain for the ratio of the coefficients the values 5.10^{-7}, 6.10^{-6}, and 2.10^{-9} respectively.

The above expression for $\frac{s}{k}$ may also be written $A \frac{N'}{N} \nu'$, where A only involves λ, and is independent of the physical condition of

* *Amsterdam Dissertation*, 1902; also *Arch. Néerl.* (2) x. 148.
† *Annalen d. Phys. und Chem.*, xxiii. 758.
‡ *Berichte d. Phys. Gesell.*, x. 858.

the medium. Thus the ratio $\frac{s}{k}$ may be increased by increasing $\frac{N'}{N}$, or by decreasing ν'. In the experiments whose results have been used above, $\frac{N'}{N}$ was a fraction varying from 10^{-2} to 2.10^{-5}. Now the number of electrons per molecule which can affect anomalous dispersion at a given line is of the order of the valency of the element.* Thus the maximum value of $\frac{N'}{N}$, which can only be attained under ideal conditions, is not more than a few units. Again ν' is in general regarded as slowly increasing with density, though Ladenburg expresses doubt as to the direction in which ν' varies when the density is increased or decreased.

If in the experiments quoted above we imagine the condition of the vapour to be so varied that $\frac{N'}{N}$ becomes unity while ν' remains unaltered, the ratio $\frac{s}{k}$ takes the values 10^{-4}, 6.10^{-4}, and 10^{-4} for Hallo's, Geiger's, and Ladenburg and Loria's medium respectively. It thus appears probable that under the conditions existing in the solar atmosphere $\frac{s}{k}$ is always a very small fraction.

Thus the attenuation produced by anomalous scattering, regarded as following Rayleigh's law, will only be a very small fraction of that produced by absorption, and the scattering effect will be masked by the absorption line. Rayleigh's formula only gives the attenuation of the original beam, while in the case of the Sun, where the radiation comes from an extended surface, a certain amount of light originating at different points of the disc will be scattered into the direction of the observed beam. Thus the effective attenuation by scattering will be less than that given by Rayleigh's formula.

§ 5. Fraunhofer Lines.

We shall consider the factors which enter into the formation of the lines in the spectrum of any small portion of the Sun's disc, excluding from the present discussion those lines which are reversed or doubly reversed, like H and K. At least five factors must be considered.

1. True absorption by the superincumbent atmosphere.

2. Pressure of the solar atmosphere. This produces a slight displacement to the red.

3. Anomalous refraction of those rays originating in the given area whose wave-lengths differ but slightly from that of a true absorption line. These rays are subject to greater curving in the solar atmosphere, and may be regarded as "scattered by refraction."

* *Vide* J. J. Thomson, *Corpuscular Theory of Matter.*

The effect will be slightly greater for R-light than for V-light, on account of the dissymmetry of the dispersion curve.

4. Rays originating outside the given area may be anomalously refracted in such a way as to reach the observer along the line of vision of the given area. This factor, when it enters, will tend to destroy the dark wings produced by the third factor.

5. Anomalous scattering of light by the molecules of the solar atmosphere. It has already been shown that, if this factor can be represented by Rayleigh's law, the effect will be masked by the true absorption effect. The attenuation due to scattering will not be as great as that demanded by Rayleigh's law, since a portion of the radiation from other parts of the photosphere will be scattered into the direction of the observed beam. Thus this factor may be neglected in comparison with the true absorption effect.

As we go from the centre of the disc towards the limb, the absorption effect is increased. The displacement due to pressure is also increased. The displacements which the Fraunhofer lines show at the limb, relative to their positions at the centre of the disc, are generally attributed to the increased pressure effect, but Julius* attributes them chiefly to the effect of anomalous refraction and anomalous scattering. But since the scattering effect is in general negligible, we need only consider the effects of anomalous refraction, included in the 3rd and 4th factors given above. These factors must be considered for three separate cases, according to the nature of the density gradients assumed to exist in the solar atmosphere. We might regard the density of the solar atmosphere as decreasing outwards, as required by the gravitational law. On the other hand, Emden's theory would lead us to expect the formation of a large number of whirls in the Sun's atmosphere, yielding what might be called a "tubular structure." Again, we might conceive of the existence, in the solar atmosphere, of a large number of irregular density gradients, caused by convection currents, or local differences of composition or temperature. Such a state seems highly probable since the effect of gravitation is largely counteracted by other forces, such as light pressure. The three cases here enumerated will now be considered in detail.

(*a*) *Radial Density Gradient—Density Decreasing Outward.*

It can easily be seen that near the centre of the disc anomalous refraction would have only a very slight effect, and so the lines in whose neighbourhood there is anomalous refraction would have only very slight wings. Near the limb the R-light which is refracted away from the observer is compensated for by R-light from other parts of the disc undergoing anomalous refraction towards the observer, while there is no such compensation for V-light lost through anomalous refraction. Thus the lines in the spectrum of the limb should have a wide wing to the violet side,

* *Le Radium*, Oct. 1910; *Proc. Roy. Acad. Amst.*, xiii. 881; *Astrophysical Journal*, xxxi. 419.

and no wing on the red side, and should show a resultant displacement to the violet. This effect should be most marked for those lines which can show the greatest anomalous refraction under the conditions which hold in the Sun's atmosphere.

The displacements and intensities of lines at the centre and limb of the Sun have been investigated by Hale and Adams,* and by Fabry and Buisson.† At the centre of the disc the lines show a slight displacement towards the red, which is interpreted as a pressure effect. At the limb some lines are strengthened and others weakened in varying degrees, but all are displaced to the red except two or three, which show a well-marked displacement to the violet. In general, the lines which are widened at the limb are widened at the red edge only, the violet edge being undisplaced. This is directly opposed to the theory given above, which requires a displacement to the violet. Again the anomalous dispersion theory would require the lines which show the strongest anomalous dispersion effects in the Sun to be most strengthened and most displaced at the limb; but Adams finds that the lines which are most strengthened are least displaced. There appears to be no correspondence between the observational data and the phenomena which we should anticipate on the basis of anomalous refraction in a solar atmosphere whose chief density gradient is radial.

(*b*) *Tubular Structure.*

Take a point P near the centre of the Sun's disc, and suppose the solar atmosphere above this point to have a tubular structure due to the presence of a large number of whirls (fig. 1). Rays not

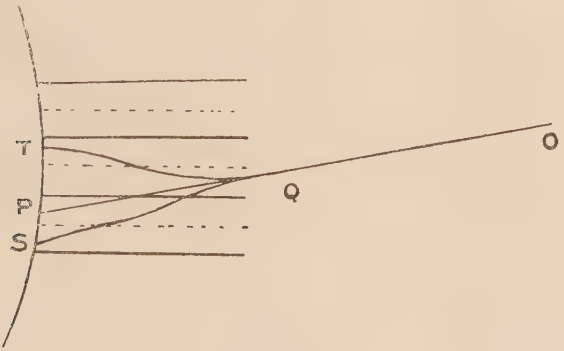

Fig. 1.

subject to anomalous refraction reach the observer along a nearly straight path PQO. R-light and V-light originating in P will be considerably curved in the tubular structure, and will in general

* *Astrophysical Journal*, xxv. 300; also W. S. Adams, *Astrophysical Journal*, xxxi. 30.

† *Astrophysical Journal*, xxxi. 97.

fail to appear in the spectrum of P. The defect will be to some extent compensated for by R-light from S, and V-light from T, which will be refracted in the tubular structure in such a way as to reach the observer along QO. Thus the lines in the spectrum of the centre of the Sun's disc should have only slight wings, and should have no appreciable displacement except the displacement due to pressure. When the point P is near the limb, rays reaching the observer will traverse the tubular structure in a direction nearly perpendicular to the lines of greatest and least density of the structure, and so will be subject to a considerable curving effect in planes perpendicular to these lines. This curving effect will be equivalent to a scattering by refraction. Further, the paths of the rays will be considerably longer at the limb than near the centre of the disc. The combined result of these two factors will be to produce a wider dispersion band than was obtained at the centre of the disc. Thus anomalous refraction in an atmosphere having a tubular structure should yield a narrow dispersion band at the centre of the disc, and a wider dispersion band, displaced to the red on account of the dissymmetry of the dispersion curve, at the limb. The lines which show anomalous dispersion effects most strongly in the Sun should show the greatest widening and the greatest displacements at the limb; but Adams * finds that the lines which show the strongest wings at the centre of the disc are almost undisplaced at the limb, while their wings are weakened, and in some cases entirely destroyed. In general the lines which are most strengthened are least displaced. The lines 4227, D_1, D_2, b_1, b_2, b_4, and the hydrogen lines all have pronounced wings at the centre of the disc. They are all lines at which anomalous dispersion has been observed in the laboratory, and their appearance in the spectrum of the centre of the Sun's disc points to their being the lines most likely to show anomalous dispersion effects in the Sun; *i.e.* they should have their wings strengthened at the limb, and should also show a pronounced displacement to the violet. But Adams found that they lost their wings, or had their wings considerably weakened in the spectrum of the Sun's limb. Thus we are forced to the conclusion that anomalous refraction in a tubular structure cannot be an important factor in the formation of the Fraunhofer lines.

(*c*) *Irregular Gradients.*

If the solar atmosphere contained a very large number of irregular density gradients, it is easily seen that the Fraunhofer lines near the centre of the disc should be winged; since the anomalously refracted rays reaching the observer would have left the photosphere in a more oblique direction than the rays which had suffered no anomalous refraction. For Julius † has shown that the amount of radiation reaching a point at the outer boundary

* *Astrophysical Journal*, xxxi. 46.
† *Proc. Roy. Acad. Amst.*, xiii. 1269, fig. 4.

of the solar atmosphere varies considerably with direction, being greatest in a direction normal to the photosphere. This deduction is founded on observations of the distribution of intensity over the Sun's disc, and is independent of any theory of the causes underlying the variation. It is clear that the average effect would be to yield a winged line, with a slight displacement to the red. In the spectrum of the limb the wings should be widened, since the paths of the rays through the region of irregular gradients is considerably increased. The widening should be greater on the red than on the violet side, and the lines should therefore have an additional displacement to the red at the limb. Both the widening and displacement should be greatest for the lines which show the greatest anomalous dispersion effects in the Sun's atmosphere. These effects are almost precisely similar to those which would hold if the solar atmosphere had a tubular structure. They have been compared with the observational data in the last paragraph, and have been found to show little or no correspondence with these data.

The most useful criterion which can be applied to test the efficiency of anomalous refraction in accounting for the variations in the form and position of the Fraunhofer lines, is the behaviour of the winged lines. No possible arrangement of density gradients appears to be capable of yielding a wide, almost undisplaced, dispersion band at the centre of the disc, and a narrow undisplaced dispersion band at the limb. It is seen that such a criterion is entirely unfavourable to the anomalous dispersion theory. We conclude that anomalous refraction in the density gradients of the solar atmosphere cannot be an important factor in the formation of the Fraunhofer lines.

§ 6. The H and K Lines and Spectroheliograph Results.

Julius* bases an explanation of spectroheliograph results on the assumption that the solar atmosphere has a tubular structure, due to the presence of a large number of whirls. This tubular structure is regarded as having the same gathering power for light as a bundle of glass tubes, and the double reversal of H and K in the solar spectrum is ascribed to its action. In some later papers Julius† considers the additional effect of anomalous scattering, and attributes the form of H and K at the centre of the Sun's disc mainly to this cause. It has already been shown (§ 4) that the effect of this factor would always be masked by the true absorption effect, and so we may omit anomalous scattering effects from the subsequent discussion.

The strongest evidence which is brought forward to support the view that the tubular structure produces the double reversals, is an experiment in which Julius obtained a double reversal of the

* *Astrophysical Journal*, xxi. 278; *Proc. Roy. Acad. Amst.*, xi. 213.
† *Proc. Roy. Acad. Amst.*, xiii. 881–897, 1263–1273.

D lines.* A beam of light from an arc lamp A (fig. 2) is passed through a long sodium flame, and the emergent beam examined by means of a spectroscope, whose slit is placed at B. The D lines are found to be doubly reversed. This reversal can be easily explained if the flame be assumed to have a tubular structure. The rays which enter the spectroscope must reach B along the direction AB, or along a direction making a small angle with AB. Starting from B in the direction BA we can draw the paths which rays of different wave-lengths must follow in order to enter the spectroscope. Rays for which $|\mu - 1|$ is small will only be slightly curved in the tubular structure, and will follow a path such as BS, which does not pass through A; while rays for which $|\mu - 1|$ is larger may be curved about in the tubular structure in such a way as to follow a path BTA. Thus the only wave-lengths missing from the spectrum of A, as viewed along the direction BA, will be those corresponding to the true absorption line, and those for

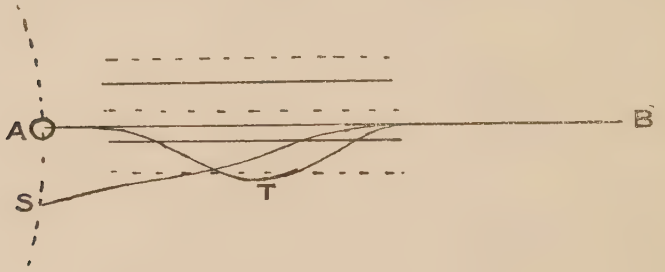

FIG. 2.

which $|\mu - 1|$ is comparatively small. The resultant effect will be to yield a doubly reversed line of a form similar to that of H and K in the solar spectrum.

But if the small source of light A be replaced by a large glowing surface such as is represented by the curved dotted line in fig. 2, even the rays which undergo very slight curving will reach the source of light. Rays of all wave-lengths, except those which undergo absorption in the flame, will then pass into the spectroscope, and the resulting line should be a dark absorption line, possibly flanked by faint wings due to anomalous refraction. The form of the H and K lines should not differ materially from that of the other Fraunhofer lines. There appears to be no ground for expecting the tubular structure to yield a double discontinuity in the spectrum. We have seen (§ 5) that the only effect which the tubular structure can have at the limb is to produce a widened line slightly displaced to the red. But H and K are found to be doubly reversed on all parts of the Sun's disc, though the reversal is difficult to photograph near the centre of the disc. If our reasoning is accurate, the anomalous refraction theory fails to account for this phenomenon.

* *Arch. Néerl.* (2) viii. 394.

More recently Julius* has considered the influence of an atmosphere containing a large number of irregular density gradients on the form of H and K. We have seen that at the centre of the disc the only possible effect of anomalous refraction in such an atmosphere would be to yield a true absorption line, with dark wings due to anomalous refraction. There is thus no reason whatsoever for supposing that anomalous refraction could yield a bright reversal of a line in the spectrum of the centre of the disc. In order to consider the possibility of the reversal of the K line at the limb by anomalous refraction in irregular density gradients it is necessary to take into account Julius's "irradiation curve" at a point M near the outer limit of the solar atmosphere (fig. 3). Suppose an observer at O views M from such a direction that M

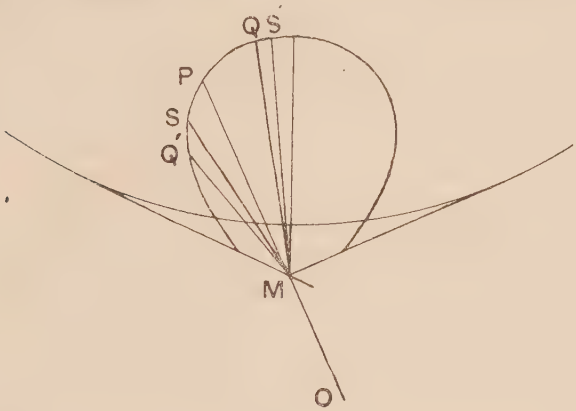

Fig. 3.

appears some distance away from the centre of the disc. Suppose also that at M there is a density gradient whose radial component is directed inwards; then it sends to the observer R-light leaving the photosphere in some such direction as MQ, and V-light leaving the photosphere in some such direction as MS. The intensities of R-light, V-light, and continuous spectrum reaching the observer from the direction MO will be represented by MQ, MS, and MP respectively, and so there may appear in the spectrum the red component of K_2. In order to account for the simultaneous appearance in the spectrum of the violet component of K_2, it is necessary to suppose that quite near to M there is another density gradient oppositely directed to the first, and so sending towards the observer R-light from the direction MQ′, and V-light from the direction MS′. The resultant intensities of R-light, V-light, and the continuous spectrum, when the joint effect of the two density gradients are considered, are MQ + MQ′, MS + MS′, and 2MP,

* Vide *Handwörterbuch der Naturwissenschaft*, vii. 824, article "Physik der Sonne," in which Julius gives an account of the whole anomalous dispersion theory of solar phenomena.

respectively. There is thus no reason for supposing that the resultant intensity of the light undergoing considerable curving in the density gradients should be greater than that of the light which undergoes only slight curving. Hence there is no reason for expecting that an atmosphere containing a large number of irregular density gradients should yield a doubly reversed K line, as a result of anomalous refraction.

It is found that over the brightest regions on spectroheliograms, or over the flocculi, the K line has a double bright reversal whether the flocculi be viewed at the centre of the disc or at the limb.* On some of Deslandres' vitesse plates † there appear extensive regions, widely scattered over the disc, where K_2 is unusually strong and symmetrical. It is only in a few isolated regions that only one component of K_2 appears. We have already seen that this symmetrical bright reversal of K_2 cannot be explained as an anomalous refraction effect, and so the flocculi cannot be explained as anomalous refraction effects; they must be attributed to some such cause as increased radiation. But even over other regions than the bright flocculi, the K_2 line appears quite distinct, and in general it is symmetrical or nearly so, both components being visible except in small isolated regions. This form of the K_2 line cannot be accounted for as an anomalous refraction effect. Julius attributes the faint appearance of K_2 over the general disc to anomalous scattering; but we have seen that Rayleigh's formula does not yield a sufficiently strong scattering effect to have any appreciable effect in comparison with that of true absorption. As K_2 appears nearly symmetrical above the regions which appear bright in spectroheliograms, and as this reversal cannot be accounted for on the anomalous dispersion theory, the bright details of spectroheliograms cannot be explained as effects of anomalous refraction in the solar atmosphere. Further, if it can be granted that the very bright flocculi are due to strongly radiating clouds of calcium, there is no difficulty in supposing that the other bright details of spectroheliograms are due to increased radiation.

Flocculi have been observed to move across the Sun's disc, showing only such differences of form as might be attributed to differences in perspective. If the flocculi were due to anomalous refraction in a tubular structure, this would require that the tubular structure should be able to converge light upon any direction within a solid angle of 2π; which is equivalent to saying that the tubular structure could produce no convergence at all. If the flocculi be attributed to anomalous refraction in irregular density gradients, the persistence of the flocculi would involve the persistence of the irregular density gradients. Further, if the calcium flocculi and the hydrogen flocculi be attributed to anomalous refraction effects in the same system of density gradients, they should follow the same law of rotation. But the calcium flocculi show the same equatorial acceleration of rotation as the faculæ

* St. John, *Astrophysical Journal*, xxxii. 48.

† See, *e.g.*, Meudon, *Annales*, t. iv., 1re partie, pl. 27.

and sunspots, while the hydrogen flocculi show no equatorial acceleration.

Bevan * has suggested an explanation of the form of the K line on the Sun's disc, based on the assumption that K_1 is a true absorption band produced in the dense lower region of the solar atmosphere, while K_2 is formed by anomalous refraction in irregular density gradients in the upper regions of the atmosphere. He assumes considerable local differences in the power of transmission of the lower regions of the solar atmosphere. At points near the limb, only one component of K_2 could be produced by such a system of density gradients as Bevan suggests. On this view, the mean positions of K_2 and K_3 should be undisplaced over the whole disc. This is at variance with the observations of St John, according to which K_2 and K_3 are systematically displaced to the violet and red respectively.

The above examination points to the conclusion that anomalous refraction in the solar atmosphere cannot be an important factor in the production of the double reversal of H and K, or of the bright details of spectroheliograms.

§ 7. The Prominences.

According to Emden's theory,† the Sun is divided into strata by surfaces of discontinuity, whose general form is that of a family of hyperboloids of revolution, having the axis of the Sun as axis of revolution. The stratification is the result of the action of gravitation and rotation. The velocities on opposite sides of a surface may differ considerably; and this causes friction, which tends to cause waves to form in the surface. The waves increase in amplitude and finally break, forming whirls. The process is somewhat similar to that underlying the formation of the "mackerel-back sky." At points on a given surface of discontinuity the difference in the velocities on opposites sides of the surface will vary inversely as the perpendicular distance from the axis of the Sun.‡ We might thus expect the breaking of the waves to begin at points near the axis of revolution, and to proceed outwards to the photosphere, in much the same way as the breaking of a wave proceeds along the seashore.

Julius§ combines Emden's theory with that of Schmidt, according to which the Sun is a mass of gas having no clearly-defined boundary, while the photosphere, the apparent boundary, is purely the result of refraction. Taking this into consideration, Julius regards the whirls which form in the surfaces of discontinuity as proceeding outwards to a considerable distance beyond the photosphere, and so giving rise to the prominences. He regards the visibility of the prominences as due, not to selective emission of

* *Monthly Notices*, November 1911.

† Emden, *Gaskügeln*, p. 429; *Sitzungsberichte der k. b. Akad. Wiss.*, München, 1901, Heft 3; *Astrophysical Journal*, i. 112.

‡ *Gaskügeln*, p. 432.

§ *Proc. Roy. Acad. Amst.*, v. 162.

the vapours in the whirl, but to anomalous refraction of rays emanating from the photosphere, in the density gradients of the whirl.

One feature of this theory is, that it explains the apparent outward motion of the prominences without supposing it to be an actual transmission of matter. For this apparent motion may now be regarded as the transmission of a condition, that of the breaking of a wave to form a whirl. This would explain without difficulty the very high and arbitrarily varying velocities of ascent of prominences recorded by observers of solar phenomena, without assuming any actual outward transmission of matter. In the light of purely gravitational theory it is difficult to explain the very high velocities of ascent of some prominences, sometimes amounting to 100 km./sec. This difficulty would disappear if we could regard the outward moving prominence as a whirl in the process of formation.

One of the outstanding difficulties of the whirl theory of prominences lies in the fact that it involves the permanent presence of matter at the greatest heights above the photosphere to which prominences have been traced. For the density gradient within the whirl would be formed by a horizontal redistribution of matter through the region occupied by the whirl.

The earlier measurements of the apparent upward velocities of the prominences were made visually at the limb. Recently St John * and Deslandres † have found upon their "vitesse plates" an additional absorption line, generally upon the violet component of K_2 or to the violet side of that component, with a displacement amounting to 15–100 km./sec., when interpreted as a radial velocity effect. Thus the extra absorption line shows that there are clouds of comparatively cool vapours moving upward through the solar atmosphere with velocities of the same order of magnitude as the upward velocities of prominences measured at the Sun's limb. It seems highly probable that these clouds are the prominences. Now the whirl theory does not require an actual motion of transmission of the prominences, and so we might regard the evidence of the vitesse plates as unfavourable to the theory. On the other hand, we might regard the vapours which register their motions by the additional absorption line as proving the existence of material for a more extensive atmosphere, in which anomalous refraction might have an appreciable effect.

If a prominence seen at the Sun's limb were due to anomalous refraction in a whirl formed in a surface of discontinuity, we should expect it to have a simple columnar form, but in the prominences registered by the spectroheliograph complete arches and plumes are much more frequent. The mechanism provided by Emden's surfaces of discontinuity does not appear to be capable of explaining these complicated forms.

We can calculate the greatest height at which it is *geometrically* possible for a whirl with a given zenith distance to be made

* *Astrophysical Journal*, xxxiv. 132.

† *Meudon, Annales*, t. iv., 1re partie, plates 46, 47.

visible by anomalous refraction. The *physical* possibility will depend upon the magnitude of the density gradient in the whirl, and of this we know nothing. As a whirl is symmetrical about its axis, the incident and emergent rays make the same angle with this axis; and so the effect of the whirl upon the direction of a ray is equivalent to a rotation about the axis. In fig. 4 let the circle represent the Sun's limb, the observer being situated on the normal through C to the plane of the paper. Let PQS be the direction of the axis of the whirl, and let CPN be a radius of the circle. $\angle$ QPN is the zenith distance of the whirl. The axis of the whirl should be in a plane passing through the axis of the Sun. The axis of the Sun makes an angle of 83° with the plane of the ecliptic, and so the greatest angle between the axis of the Sun and the plane of the disc is 7°. As an approximation we may consider the axis of the Sun, and therefore the axis of the whirl, as lying in the plane of the limb. If the axis of the whirl be normal to the

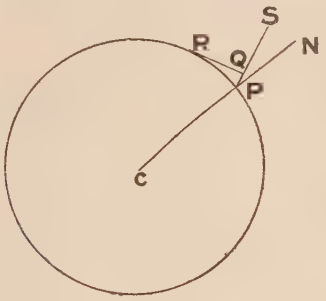

Fig. 4.

limb, it can easily be seen that the whirl cannot refract rays originating in the photosphere in such a way as to reach the observer. For the direction of such a ray would have to make an angle of less than 90° with PN, the angle being measured from PN. When the whirl has a zenith distance z, *i.e.* when $\angle$ QPN $= z$, a ray originating in the photosphere at any point R, and passing through the whirl at Q, may be refracted in the whirl, so as to reach the observer if $\angle$ RQS be equal to the angle between PQ and the line of vision of Q. But $\angle$ RQS is obviously least when R is on the limb, and so rays originating on the limb itself have the best chance of being anomalously refracted so as to reach the observer. The highest possible position of Q is reached when QR is tangential to the limb, subject to the condition given above as to the value of $\angle$ RQS. For the purposes of this discussion we shall suppose that the path of the ray between R and Q, and between Q and the observer, consists of two straight lines, *i.e.* we suppose that only the density gradients in the whirl are sufficient to produce an appreciable curving of the ray. Take as rectangular co-ordinates the tangent to the circle at P, the radius PC, and the normal through P to the plane of the limb. The direction cosines of the

line of vision of P are 0, sin 16′, cos 16′, while those of PQ are $-\sin z$, $\cos z$, 0. Thus the angle between PQ and the line of vision of $P = \cos^{-1}(\cos z \,.\, \sin 16') = 90^\circ - 16' \cos z$, approximately. Similarly the angle between PQ and the line of vision of Q when Q is x'' above the limb, $= 90^\circ - \left(16 + \frac{x}{60}\right)' . \cos z' . = \mathrm{PQO}$. Here z' is the zenith distance of the whirl at Q, and O is the position of the observer above the plane of the paper. $\angle$ RQS must not exceed this minimum value.

$$\mathrm{Sec\ RCQ} = 1 + \cdot 001\ x. \quad \mathrm{RQC} = 90^\circ - \mathrm{RCQ}. \quad \mathrm{RQP} = \mathrm{RQC} + z.$$
$$\mathrm{RQS} = \mathrm{RCQ} + 90^\circ - z.$$

Thus the condition to be satisfied at the highest possible position of Q is

$$\mathrm{RCQ} + 90^\circ - z' = 90^\circ - 16'(1 + \cdot 001x) \cos z'.$$

A table of values of x for given values of z' can be calculated from this formula. The results are given in Table V.

TABLE V.

z'	x	Observed Heights.
°	″	″
10	14	661
15	34	222
20	62	93
30	152	60, 102, 196
40	300	228, 340

The last column gives the heights of prominences with the given zenith distances, taken from photographs by Fényi, Slocum, and others, reproduced in the *Astrophysical Journal*, for cases where the prominences appear to have a simple columnar form. The last column shows that in general the prominences extend much higher than the theory would permit. The preceding calculations are based on the assumption that the density gradient in the whirl is sufficient to turn the ray from the direction RQ to a direction nearly perpendicular to it. As nothing is known of the magnitude of the density gradients, it is impossible to judge whether this assumption is justified or not. But the geometrical treatment to which we have subjected the theory is unfavourable to the explanation of prominences by anomalous refraction in whirls of the nature here considered.

Emden's theory, as extended by Julius to the prominences, ascribes both sunspots and prominences to the formation of whirls in the surfaces of discontinuity. Emden shows that such whirls would be formed with difficulty near the equator and near the poles, and so the sunspots and prominences should both be restricted to zones of medium latitude. We could also infer that the prominences should display the 11-year sunspot period. A

comparison of the prominence and sunspot activity through a complete 11-year period is not quite so favourable to the new view. According to W. J. S. Lockyer,* the manifestations of prominence and sunspot phenomena are almost exactly similar, except for a difference of several years in their time of maxima. They behave as though they were different aspects of one phenomenon, with a difference in phase of several years. But if a prominence is a whirl whose formation proceeds rapidly outwards from the photosphere, and which gives rise to a sunspot at the level of the photosphere, the difference of phase between the two forms of solar activity should be inappreciable, since the mechanism producing the two is the same.

We can easily see that a whirl may transmit to the observer rays for which $\mu > 1$ and rays for which $\mu < 1$. The projections of the paths of these rays upon a plane perpendicular to the axis O of the whirl are shown in fig. 5. The density in a whirl decreases

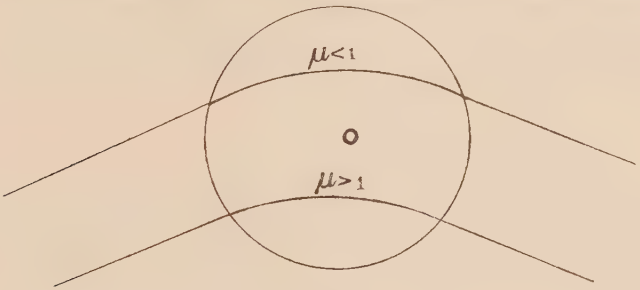

Fig. 5.

towards the axis. Thus the whirl should yield a bright line in an almost undisplaced position. But now there arises the question of the nearness to the centre of the absorption line to which anomalous refraction proceeds. We should expect a certain amount of absorption to take place, and so there should be a dark reversal upon the bright prominence lines. For anomalous dispersion and absorption require the fulfilment of the same condition, viz. the presence in the medium of electron systems which can vibrate with a given free period. The question to be decided is whether the dark absorption line will be wide enough to be detected by observation. The general theory of dispersion and absorption can yield no help in the consideration of this question. For the width of an absorption line is a multiple of ν' whose magnitude depends upon the length of path through the medium, and upon the magnitude of the coefficient of absorption in that medium. In the case of the prominences we do not know anything concerning the magnitude of this coefficient, and could only make the roughest estimate of the length of path in the medium. Hence we cannot evaluate the width of the dark reversal which should be present

* *Monthly Notices*, lxiii. 452.

upon the bright prominence line, and it is impossible to state whether a dark reversal should be detectable.

If a mass of rising vapour in the solar atmosphere contained a large density gradient, it might anomalously refract R- or V-light emanating from the photosphere, in such a way as to reach the observer. Such rays would appear in the spectrum of the prominence to the red or violet side of the centre of the true emission line, according as the density decreased or increased outwards in the prominence. Even if the prominence spectrum were mainly due to selective emission, such a process as is outlined above might enter as a subsidiary factor, yielding a widened displaced line in the spectrum. This may account for the widenings and shifts of prominence lines frequently noted by Fényi and other observers. It is impossible to judge how far such a process may be normally at work in the formation of the prominence spectrum, since a complete series of determinations of the widths and wave-lengths of lines in prominence spectra is not available.

On the whole, the above discussion is unfavourable to the anomalous refraction theory of prominences: but it must be admitted that no definitive arithmetical criterion could be found.

§ 8. The Chromosphere.

It has been suggested by Julius * that chromospheric light is not due to selective emission, but is photospheric light which has undergone anomalous refraction in the solar atmosphere. The effect of this anomalous refraction is to make the light appear to originate at some distance above the photosphere. As the nature of the effect produced by anomalous refraction will depend very largely upon the nature of the density gradients present in the solar atmosphere, it is necessary to consider the effect of different arrangements of the density gradients. We may divide these density gradients into three classes:—1st, a continuous density gradient, which may be treated as though directed along the Sun's radius; 2nd, a condensation or rarefaction in which the density gradients are arranged radially about a point; or a whirl, in which the density gradients are arranged cylindrically about an axis; 3rd, a large number of irregular density gradients.

For the case of a continuous density gradient we shall first suppose the density to decrease outwards. If a beam of light leaves P along the direction PQ (fig. 6), rays not subject to anomalous refraction will travel along a nearly straight path; but rays whose wave-lengths differ but slightly from that of the absorption line will undergo considerable curving, R-light following paths of the form PR, and V-light paths of the form PV. It is clear that only R-light could reach O in such a direction as to appear to originate above the photosphere; and so the corresponding line in the chromospheric spectrum should be wholly to the red side of

* *Proc. Roy. Acad. Amst.*, ii. 575, iii. 195, xii. 266, 446; *Physikalische Zeitschrift*, iv. 132.

the centre of the absorption line. Similarly if the density in the atmosphere increased outwards, only V-light would appear in the chromospheric spectrum, and the lines should lie wholly to the violet side of the centre of the absorption line. In either case, the lines in the chromospheric spectrum should be displaced by an amount equal to at least half their width.

According to the measurements of Hale and Adams,* the strongest chromospheric lines have widths varying from 0·07 to 0·16 A.U. The average displacement amounts to ±0·013 A.U., when considered independently of sign, and the systematic displacement is +0·002 A.U., which is less than the probable error of the observations. These measurements do not agree with what we anticipated above, on the basis of anomalous refraction in continuous density gradients. If the anomalous refraction effect here considered were superposed on the effect of selective emission,

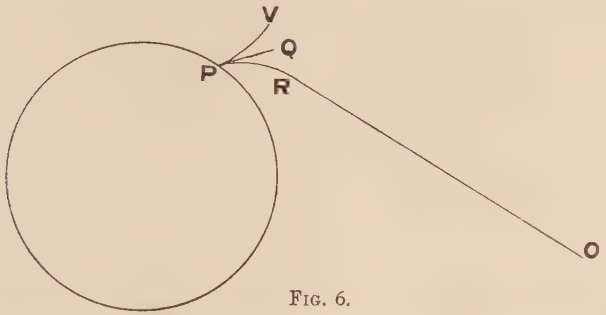

Fig. 6.

the resulting lines in the spectrum should have a systematic displacement to the red or violet according as the density in the solar atmosphere decreased or increased outwards along the radius. But according to Hale and Adams's observations, we are not justified in assuming any systematic displacement of the chromospheric lines. It thus appears that anomalous refraction in density gradients of the kind considered above cannot be an important factor in the formation of the spectrum of the chromosphere.

Julius † has shown that if the solar atmosphere contained a very large number of condensations or rarefactions, anomalous refraction could give rise to a chromospheric spectrum in which the bright lines should be only very slightly displaced to the red on account of the dissymmetry of the dispersion curve. This agrees perfectly with the measurements of Hale and Adams; but there is no obvious mechanism for supplying the large number of condensations and rarefactions necessary to the theory.

We have already examined the effect of whirls upon photospheric light. So long as the axes of the whirls are not directed normally to the photosphere, the whirls can refract photospheric

* *Astrophysical Journal*, xxx. 222.
† *Proc. Roy. Acad. Amst.*, xii. 446.

light to the observer. We have seen that it is geometrically possible for whirls with small zenith distances to refract photospheric light to the observer, up to a height as great as the height of the chromosphere. The problem of the mode of formation of a sufficient supply of whirls is solved by Emden's theory. Since a whirl is able to refract both R-light and V-light to the observer, the resulting lines in the spectrum should be very slightly displaced to the red on account of the dissymmetry of the dispersion curve. This is in perfect agreement with the observations of Hale and Adams quoted above.

If the solar atmosphere contained a large number of irregular density gradients, the effect of any one isolated gradient would be to refract towards the observer either R-light or V-light, according to the direction of the gradient, but the average effect would be to yield bright chromospheric lines, slightly displaced to the red on account of the dissymmetry of the dispersion curve. This is also in agreement with the observations of Hale and Adams.

But now there arises, as in the case of the prominences, the question whether there should be a dark reversal upon the bright chromospheric lines, if these lines are entirely due to anomalous refraction. As in the previous case, theoretical considerations cannot decide whether the central absorption line should be wide enough to be detected. No dark reversal has ever been observed, though considerable interest has been taken in the problem by a number of eclipse observers. This is not a fatal objection to the theory, as it is possible that the reversal is too narrow to be detected with the resolving power and dispersion which have been used.

Later measurements at Mount Wilson seem to point to the wave-lengths in the chromosphere being identical with those at the limb. If this be true, then it is difficult to account for the displacements at the limb as pressure effects, since it is then necessary to suppose an equal pressure in the chromosphere, which is generally supposed to be at a much higher level than the region which produces the Fraunhofer lines at the limb. The same objection does not apply to the anomalous refraction theory, since on this theory the chromospheric light is just the light which is lost at the limb on account of scattering by refraction. Thus the anomalous refraction theory would require the wave-lengths of the chromosphere lines to be identical with those of the limb lines. It should, however, be remembered that the observed shifts are very small and difficult to observe.

If the chromosphere be due to anomalous refraction in whirls formed as suggested by Emden's theory, it should be largely restricted to the sunspot zones. This does not appear to be the case. Evershed observed the 1900 eclipse at the limit of the zone of totality. The inner contacts took place, one at the N. pole of the Sun, and the other in latitude 51°, and Evershed found that the reversing layer showed the same main features in the two regions of contact.*

* *Phil. Trans.*, June 28, 1900: "Eclipse Reports."

Most of the features of the chromospheric spectrum can be accounted for on the basis of anomalous refraction, but no definitive criterion could be found to decide between this theory and the selective emission theory.

§ 9. Sunspots.

Emden * has suggested that sunspots are whirls formed in the surfaces of discontinuity of the Sun. He has shown that whirls will generally be formed in the surfaces of discontinuity at points within the body of the Sun, and that they work outwards, eventually reaching the photosphere. He contends that they give rise to sunspot phenomena when they reach the level of the photosphere, and explains the darkness of the spot by the decrease in radiation due to the fall of temperature within the whirl.

Before this theory can be considered in detail, it is necessary to examine the distribution of density and temperature within the whirl. We shall assume, 1st, the existence of a velocity potential; and 2nd, that the whirl satisfies the adiabatic law. Let p, ρ, T denote the pressure, density, and absolute temperature at a point distant r from the axis of the whirl. Let V denote the velocity at this point in a plane perpendicular to the axis of the whirl. Let suffixes o denote the values of these quantities at the outer limit of the whirl. Bernouilli's dynamical equation gives

$$\frac{1}{2}V^2 + \int\frac{dp}{\rho} = \text{constant} \quad . \quad . \quad . \quad (1)$$

From the existence of a velocity potential we have

$$Vr = \text{constant} = c \quad . \quad . \quad . \quad . \quad (2)$$

For an ideal gas $$p = \sigma\rho T \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where σ is a constant.

From the adiabatic condition, we have

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^k = \left(\frac{T}{T_0}\right)^{\frac{k}{k-1}} \quad . \quad . \quad . \quad (4)$$

From (1) and (2) $$\frac{1}{\rho}\frac{dp}{dr} = -V\frac{dV}{dr} = \frac{c^2}{r^3} \quad . \quad . \quad . \quad . \quad (5)$$

From (4) $$\frac{1}{p_0}\frac{dp}{dr} = \frac{\rho^{k-1}}{\rho_0{}^k}\frac{d\rho}{dr}.$$

$$\therefore \quad \frac{p_0}{\rho_0}k\rho^{-(2-k)}\frac{d\rho}{dr} = \frac{c^2}{r^3}.$$

Integrating this, we find

$$\left(\frac{\rho}{\rho_0}\right)^{k-1} - 1 = A\left(\frac{r_0{}^2}{r^2} - 1\right) \quad . \quad . \quad . \quad (6)$$

where $$A = \frac{c^2(k-1)\rho_0 r_0{}^2}{2p_0} = \text{constant}.$$

* *Gaskügeln*, p. 442.

Figure 7 gives the form of $\left(\frac{\rho}{\rho_0}\right)^{k-1}$. $k-1=\frac{2}{3}$, or $\frac{2}{5}$, according as the gas is mon- or di-atomic.

Since $\left(\frac{\rho}{\rho_0}\right)^{k-1}=\left(\frac{p}{p_0}\right)^{\frac{k-1}{k}}=\frac{T}{T_0}$, it is clear that the curve in figure 7 represents the general form of $\frac{\rho}{\rho_0}$, $\frac{p}{p_0}$, or $\frac{T}{T_0}$, in terms of $\frac{r}{r_0}$. The general form of the curve is not affected by the value of A. The theory is in good agreement with the measurements of density, pressure, temperature, and velocity made during cyclones.*

The density, pressure, and temperature fall at first slowly, and then rapidly, as we approach the axis of the whirl. The regions of slow and rapid fall of temperature give rise to the penumbra and umbra respectively. They are separated by a region where the gradients of temperature and density undergo very rapid changes.

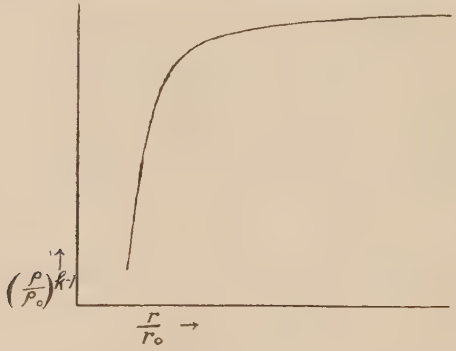

FIG. 7.

The width of this region should be inappreciable in a sunspot, and so the umbra and penumbra should appear to be separated by a sharply defined line. On this theory the restriction of the sunspots to zones of mean latitude is explained by the fact that whirls should in general be formed with difficulty near the equator and near the poles.

Julius † also ascribes the sunspots to the effects of whirls formed in the surfaces of discontinuity; but he accounts for the darkness of the spot by the scattering effect of refraction in the density gradients of the whirl, and not by a decrease in radiation due to a fall of temperature within the whirl. We have seen that the density gradients may be considerable in that part of the whirl which we have supposed to correspond to the umbra. Even the rays not subject to anomalous refraction will suffer considerable curving in passing through this region, and so the effect of the large density gradient will be to scatter the rays; so this region of

* For curves see Hann, *Meteorologie*, p. 445; *Gasküge1n*, p. 373.

† *Proc. Roy. Acad. Amst.*, ii. 575–588, xii. 266–284; *Revue générale des Sciences*, xv. 488.

the whirl should appear darker than the surrounding photosphere. The same should occur, in a less degree, in the region of smaller density gradient, or the penumbra. Rays subject to anomalous dispersion should suffer still greater scattering, and so the Fraunhofer lines in whose neighbourhood anomalous dispersion can occur should appear winged and strengthened in the sunspot spectrum.

Julius * constructed a system of gradients of optical density which were equivalent to a rarefaction or condensation by floating drops of a liquid in another liquid of equal density but of different refractive index. He found that these systems showed a darkening effect round the edges of the drops, where the gradient of optical density existed. Now a whirl is equivalent to a rarefaction, and so it should show the darkening effect at the edges, and still more in the inner region where the density gradient is greater; it should therefore yield a phenomenon similar to a sunspot.

On this theory the absorption lines in whose neighbourhood anomalous dispersion can occur, should appear widened, or winged, in the sunspot spectrum. This may be tested by a comparison of the changes which lines undergo in the spot spectrum, with Geisler's † measurements of anomalous dispersion in metallic vapours. Excluding H and K, 66 of the lines at which Geisler measured anomalous dispersion are present in the sunspot spectrum. Of these, 35 are strengthened, 2 suffer no appreciable change, and 29 are weakened. But on the above theory all the lines which show anomalous dispersion should be winged and strengthened in the spot spectrum; and so the observational data do not fit in with the theory. There does not appear to be any correspondence between anomalous dispersion data and the changes which lines undergo in sunspots.

The lines which are most widened in sunspots should be those which, under the conditions to which the vapours in the solar atmosphere are subjected, show the greatest anomalous dispersion effects. But if the chromosphere be ascribed to anomalous refraction, these lines should also appear strongest in the chromospheric spectrum. In order to compare these phenomena the 56 strongest lines (excluding H and K) were taken from Lockyer's ‡ list of chromospheric lines. These included all the lines which, on Lockyer's scale, have intensities 5–10. Of these lines 41 are weakened, 8 are strengthened, while 7 suffer no appreciable change, in the spot spectrum.

The very marked weakening of the enhanced lines in the sunspot spectrum, together with the occurrence of bands due to the oxide of titanium, and bands due to the hydrides of calcium and magnesium, seem to indicate that the temperature of the sunspot is lower than that of the photosphere. But if this be granted,

* *Proc. Roy. Acad. Amst.*, xii. 266.

† Geisler, *Zur Anomalen Dispersion in Metalldämpfen*: *Bonn Dissertation*, 1909.

‡ "Report of 1898 Viziadrug Eclipse Expedition," *Phil. Trans.*, 197A.

most of the changes which lines show in the spot spectrum can be explained as temperature effects. For Hale, Gale, and Adams * have shown that the changes in the appearance of the spectral lines, noted in going from the normal solar spectrum to the sunspot spectrum, are similar to those noted in going from the spark to the arc, from the strong arc to the weak arc, or from the core to the flame of the arc. Thus the changes noted may be ascribed to the lower temperature of the sunspots. The agreement between the laboratory results and the sunspot observations is strikingly complete, in contrast with the lack of definite correspondence between Geisler's measurements and the changes noted in the sunspot spectrum. The assumption of lower temperature would account for the shift of the maximum intensity towards the red in the spot spectrum; it would also account for the proportionately greater darkening effect at the violet end of the spectrum observed by Schwarzschild and Villiger.

The presence of magnetic fields in sunspots would account for the doublets and triplets which appear in the spectrum. Hale † measured the strength of the fields in a number of sunspots, and found that in some cases they amounted to as much as 5000 Gauss. It should be noted that Emden's theory provides a simple explanation of the existence of these fields. For it is probable that the solar atmosphere is to a certain extent ionised, and that there are electric charges present in the spots. The circular motion of these charges in the whirl would give rise to a magnetic field.

The effects of temperature and pressure, added to the Zeeman effect, seem to afford a more hopeful way of explaining the phenomena of the sunspot spectrum than does anomalous refraction.

§ 10. The 11-Year Sunspot Period.

Julius ‡ attributes the number of sunspots visible at any time to the position of the line joining the centres of the Earth and Sun relative to the body of the Sun. This joining line will trace out on the surface of the Sun a complicated spiral, whose period will be a sunspot period. According to Julius's calculations this period is 11·17 years, a value which agrees very closely with Wolf's mean sunspot period of 11·124 years. For the purposes of the calculation of the period of the spiral, it is necessary to know the synodic period of rotation of the Sun. But as different zones of the Sun rotate at different rates, it is difficult to assign a value to the synodic period. Julius gets over this difficulty by assuming for this period the mean value of 26 periods of magnetic and meteorological elements, which vary from 25·47 days to 26·67 days, having a mean of 25·924 days. The Sun is then regarded as being replaced by a sphere of equal radius, rotating as a rigid body with a synodic period of 25·924 days.

* *Astrophysical Journal*, xxv. 75, xxiv. 185, xxix. 76, xxx. 86.
† *Ibid.*, xxviii. 100.
‡ *Revue générale des Sciences*, xv. 491.

In fig. 8 let QQ′ and EE′ represent the intersections of the Sun's equator and the ecliptic, with the sphere representing the Sun. Let P be the point of intersection of the line of centres of the Earth and Sun with the sphere. Then the point P traces out a complicated spiral upon the sphere, and it is the period of this spiral which we have to calculate. Let P start from the node P_0 at the instant when the meridian M is passing through the node. At the end of a synodic rotation P will be at P_1 on the ecliptic, and the meridian M will have moved to M′. The path of P during the interval is represented by the dotted line leaving P_0 and ending at P_1. Similarly M″ and P_2 denote the positions of M and P at the end of the second synodic rotation. The positions of P_1 and P_2 are represented by the points 1, 2, on the meridian M in

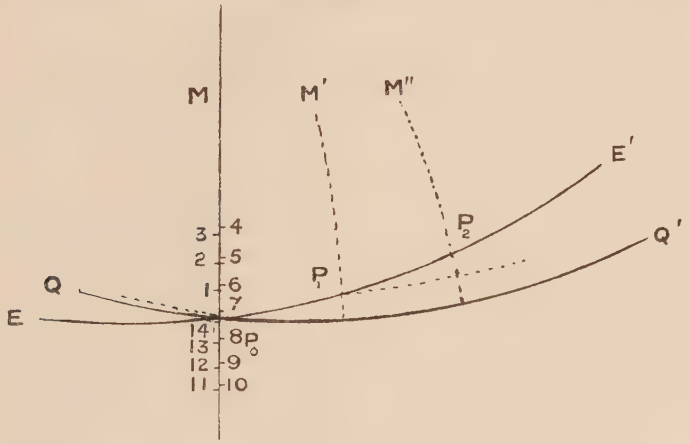

Fig. 8.

its undisplaced position. Similarly the points 3, 4 . . . 14 represent the projections upon M of the position of P after 3, 4 . . . 14 synodic rotations. The point 14 is very near to P_0, since 25·924 × 14 days = 362·936 days, which only differs from the sidereal year by 2·32 days. At the end of a year from the time it left P_0, the point P will again pass through the node, but it will not be upon the meridian M at that instant. To obtain the period of the spiral Julius divides 25·924 by 2·32, and takes the resulting quotient 11·17 to represent the number of years in the period of the spiral. But it can easily be seen that the number thus obtained can only have significance when it is exactly, or very nearly exactly, a whole number. For what is required is the interval in which P will return to the node P_0 and the meridian M simultaneously. This interval will be the least number of whole years, which is an exact multiple of 25·924 days. But 11·17 years = 11 years + 62 days = 25·924 days × 157 + 10·37 days. From this it is seen that in 11·17 years the point P will

not be back at the node, nor upon the meridian M from which it started.

To find the smallest number of years which is an exact multiple of 25·924 days, we must express $\frac{365{\cdot}256}{25{\cdot}924}$ as a continued fraction, and evaluate the successive convergents until we find one which represents the fraction with a high degree of accuracy. The denominator of this fraction will represent the required period in years. Or we can see directly from the fact that $11{\cdot}17 \times 6 = 67{\cdot}02$ that 67 years will be very nearly an accurate period for the spiral. Thus the method applied above yields a period of 67 years, and not one of 11·17 years.

It is of some interest to compare the period deduced above with the period obtained if we assume other values for the synodic period of rotation of the Sun. The mean period of the Greenwich magnetic observations, 25·86 days, yields a period of 8 years for the spiral, while the Singapore meteorological observations of Broun, which have a period of 25·83 days, yield a period of 7 years for the spiral. The sidereal period of 25·38 days, corresponding to a synodic period of 25·72 days, also yields a period of 7 years for the spiral.

In the attempted explanation of the sunspot period described above, the important factor is the return of the line of centres of the Earth and Sun to the same position relative to the actual *body* of the Sun within the given period. This appears to be equivalent to the assumption of the existence of a fixed area of the Sun's surface, whose sunspot activity far exceeds that of the rest of the disc. But if such an area existed, we should expect to find a well-defined sunspot period equal to the synodic period of rotation of the Sun, *i.e.* about 26 days. There is no evidence of the existence of such a period, though it has been searched for by Schuster and others.

In the paper already referred to,* Julius speaks of the bearing of the position of the line of centres relative to the system of surfaces of discontinuity upon the periodicity of sunspots. But the system of surfaces of discontinuity is symmetrical about the solar equator, and so the aspect of these surfaces, viewed from the Earth, depends only upon the Earth's heliocentric latitude. This latitude has a period of one year, and so the sunspots should have a well-defined period of one year, if the sunspot activity depended upon the aspect of the surfaces of discontinuity viewed from the Earth. Such a period has not been discovered, though careful search has been made for it by a number of sunspot observers.

We conclude that this form of geometrical theory does not account for the 11-year sunspot period.

* *Loc. cit.*, p. 494.

§ 11. Summary and Conclusions.

Not one of the three theories of the equilibrium of the solar atmosphere considered above can account for an atmosphere extending to the observed height of the solar atmosphere.

Even when the temperature of the photosphere is assumed to have a considerably higher value than is generally assigned to it, the angular radius of the critical sphere does not differ appreciably from the radius of the apparent photosphere.

The effect of anomalous scattering is completely masked by the true absorption effect.

The observed differences between the Fraunhofer lines at the centre and limb of the Sun do not agree with the requirements of the anomalous refraction theory, upon any hypothesis as to the arrangement of the density gradients in the solar atmosphere. In particular, the lines which have pronounced wings at the centre of the disc, which from their appearance seem most likely to show anomalous dispersion effects, do not behave as the theory demands.

It is shown that anomalous refraction in the solar atmosphere, whether that atmosphere has a tubular structure or a system of irregular density gradients, cannot account for the double bright reversal of H and K which is observed over the whole disc and particularly over the calcium flocculi. The bright details of spectroheliograms cannot be accounted for by anomalous refraction.

In dealing with the form of the Fraunhofer lines we have neglected the effect of anomalous scattering, as we have found that, with Rayleigh's form for the scattering coefficient, this factor is completely masked by the true absorption effect. Rayleigh's form of scattering coefficient was simply taken as a working hypothesis; and it is probable that the true coefficient of scattering in the neighbourhood of an absorption line has quite a different form, which may possibly yield a much greater scattering effect than Rayleigh's form. If this were so, it would affect the form of the lines showing strong anomalous dispersion, and it might possibly account for the faint bright reversal of H and K noted over the whole disc of the Sun. It could not affect the general conclusions at which we arrive in discussing the Fraunhofer lines, nor could it yield the very bright K_2 observed over the flocculi.

It is found that anomalous refraction in whirls should yield bright, almost undisplaced, lines in the prominence spectrum; while the mechanism for producing a sufficient supply of whirls would be provided by Emden's surfaces of discontinuity. The bright prominence lines, if produced by anomalous dispersion, should show dark reversals, but it cannot be decided from theoretical considerations whether these reversals should be wide enough to be detected. The discussion is on the whole unfavourable to the anomalous dispersion theory, but no definitive criterion could be found.

Anomalous refraction in whirls, or in an atmosphere containing a large number of irregular density gradients, would yield a chromospheric spectrum consisting of bright lines in a nearly

undisplaced position. The bright lines should be reversed, but, as in the case of the prominences, the reversal may be too narrow to be detected in eclipse photographs. No definitive criterion could be found, and the anomalous refraction theory of the chromosphere could be neither proved nor disproved.

Most of the changes observed in the Fraunhofer lines in the sunspot spectrum would appear to be more easily explained on the basis of a lower temperature in the sunspots, combined with the presence of a magnetic field, than as the effects of anomalous refraction in whirls. On the anomalous refraction theory the strong chromospheric lines should be strengthened in the sunspot spectrum; but this is not borne out by the observational data.

Our final position is as follows:—In the case of the chromosphere and prominences, no definitive criterion has been found to decide between the anomalous dispersion theory and the selective emission theory. In the case of all the other phenomena considered, some criteria are found, and they are in all cases unfavourable to the anomalous dispersion theory; also the geometrical discussion of the sunspot periodicity was found to be defective. As a result of this discussion of the various phenomena attributed to anomalous dispersion, it is maintained that, except possibly in the case of the chromosphere and prominences, anomalous dispersion is not an important factor in producing any of the observed solar phenomena. Much more observational data will have to be acquired before it will be possible to decide how far anomalous dispersion is a secondary factor in producing the phenomena discussed above.

In conclusion, I wish to express my indebtedness to Professor Newall for suggesting this investigation, and for his kindly interest and advice during the course of the work; and also to Professor Julius for his kindness in reading my manuscript and discussing it very thoroughly with me.

Cambridge Observatory:
1913 *June* 11.

Note added July 16.—I have to thank Professor Fowler for drawing my attention to a paper (*Astrophysical Journal*, xxiv. 82) in which Mitchell describes the observation of a large number of double chromospheric lines.

The Planet Jupiter in 1890. (Preliminary Note.)

By A. Stanley Williams.

In the course of the opposition of 1890 transits of spots to the number of 1101 were observed by the writer with a $6\frac{1}{2}$-inch reflector, and there are in addition available 115 transit observations by other observers, making a total of 1216 observations. The valuable series of observations by Professor E. E. Barnard with the 12-inch refractor of the Lick Observatory, published in

vol. li. of the *Monthly Notices*, is particularly worthy of mention, and has been very useful. The whole of this material has now been reduced and discussed in the same manner as that of the previous two oppositions.* The following little table contains a summary of the rotation periods in different latitudes.

Summary of Rotation Periods for 1890.

Current.	Locality of Spots.	Rot. Per. h m s	No. of Spots.
Northern current	North of lat. +25°	9 55 38·72	5
North tropical current	North tropical spots	9 55 37·21	18
Equatorial current	North equatorial spots	9 50 27·98	23
,, ,,	Mid-equatorial spots	9 50 26·67	5
,, ,,	South equatorial spots	9 50 28·00	18
South temperate current	South tropical spots	9 55 17·47	11
,, ,,	South temperate spots	9 55 17·20	25
Southern current	South of lat. −30°	9 55 06·7	1
Red spot current	Red spot	9 55 40·63	1
,, ,,	White spot prec. R.S.	9 55 40·26	1

Altogether 108 spots were sufficiently well observed to yield useful determinations of rotation period. The consideration of numerous measures for latitude, the physical aspect of the planet during the opposition, and above all the remarkable phenomena attending the conjunction of the South Tropical Disturbance of 1889-92 with the Red Spot, must be reserved for a future occasion. It may be mentioned here, however, that one important feature of the opposition was a general absence of conspicuous spots in the equatorial regions of the planet. Against this must be set an outburst of small but very dark and very definite spots on the north side of the North Equatorial belt, and the presence of a great many markings south of the South Equatorial belt.

20 *Hove Park Villas, Hove:*
1913 *June* 5.

On the Polar Diameter of Saturn and the Minor Axis of the Ring.
By E. E. Barnard.

Considerable interest seems to have been shown during the present opposition of Saturn as to whether the polar diameter of the planet at any time exceeded the minor axis of the ring. Some observers, at various times during this opposition, made the polar diameter larger, and others smaller, than the minor axis. This is an important question, as it may have a bearing upon any eccentricity in the form of the ring.

I have been interested in the same subject, and my observations

* See *Monthly Notices*, vol. lxxii. p. 488.

with the large telescope extend over the period 1912 July 30 to 1913 March 4. At no time during this period did the polar diameter exceed the minor axis of the ring. They possibly coincided, but when best seen, the polar diameter was just perceptibly less than the minor axis of the ring.

There was so very little contrast between the outer ring and the south limb of Saturn that good seeing was required to be certain of the exact position of the limb, when so projected. The north limb was not seen during the observations, being constantly hidden by the ring.

I have also one or two unpublished observations made in 1897 with the 12-inch telescope, aperture 10 inches.

The following are the notes that concern this question, and other aspects of the ring system. Some questions have also arisen as to the exact location of the inner edge of the Crape ring. These are answered in the notes, so far as my observations go.

1897 July 1. The polar diameter of Saturn and the minor axis of the ring exactly coincided. The coincidence was perfect on this date and also on June 29, with the same telescope.

1898 July 7. The ball perhaps extended very slightly beyond the Cassini division, north.

1901 Aug. 23. A fair estimate made the Crape ring extend 0·6 toward the ball on each side; not more than that. It was well defined.

1912 July 30. The ball was slightly less than the minor axis of the ring. It was entirely hidden by the rings on the north side.

Aug. 20. The ball seemed to be about 1″ less than the south edge of the ring. There were two rather broad belts just south of the equator. No definite polar cap, but dark at the south pole.

Sept. 7. When best seen there was nothing abnormal about the planet or rings.

Oct. 12. When best seen there was nothing abnormal about the ball or rings.

Oct. 15. Once or twice for a few minutes the seeing was fine, and the belts came out clearly. There was nothing abnormal about the planet or rings.

Oct. 29. In moments of steadiness the south limb was about $\frac{1}{2}$″ within the edge of the ring. It did not exceed $\frac{1}{4}$″ (better estimate). It was just less, and that was all. With 700 diameters.

Nov. 10. In moments of steadiness the south limb was apparently at the edge of the ring. The pole was dark. There were only momentary glimpses, and they were unsatisfactory.

Nov. 15. The south edge of the ring was perhaps $\frac{3}{4}$″ beyond the ball. It was decidedly beyond the ball, but seen only by glimpses, and estimate poor. The Crape ring extended one half way to the ball.

Nov. 16. Behind the ball, south, there was a little notch at each side where the shadow of the ball ran into the Cassini division. The shadow was seen on the inner ring on both sides,

but not on the outer ring. Seeing very poor, and there were only moments of comparative steadiness. The ball seemed to be about even with the minor axis of the ring or very slightly less, possibly $\frac{1}{2}''$ less, but the seeing would not permit it to be seen definitely. The narrow dark belt south of the equator was very heavy and apparently irregular, as if spotted, but the seeing would not permit one to decide. 460 diameters. With 700 diameters there was just a little difference between the south limb and the south edge of the ring—perhaps $\frac{1}{2}''$, the ball being very slightly less than the ring. This only in moments of comparative steadiness. The ring completely hid the ball north. The rings were a little brighter than the ball. The outer ring was very slightly less luminous than the inner ring. The outer part of the inner ring was the brightest. The Crape ring was narrow where it fringed the bright ring in crossing the ball, which was seen through it. When best seen it seemed to extend half-way to the ball at the ansæ. The south pole was dusky. The dark south polar cap was not definite at the edge.

Nov. 17. With 700 diameters. Seeing very good. Could see the south limb well. It was only slightly short of the edge of the ring—estimated $0''{\cdot}2$ or $0''{\cdot}3$ less. The Crape ring extends 0·55 or 0·60 the space from the bright ring to the ball, preceding. At the following side it reached 0·55 the distance to the ball. It was well defined at the edge and of uniform brightness—a light grey. Perhaps it was a little brighter at the following ansa. It was equally well defined at its inner and outer edges (where it joined the bright ring); there was no diffusion. The polar cap was dusky, fairly well defined, with a narrow belt north of it. There were several other belts. The interval between the south limb and the edge of the ring was from $1\frac{1}{2}$ ($0''{\cdot}13$) to 2 ($0''{\cdot}18$) times the thickness of the wire— perhaps not so much as twice the thickness. The following measures (double distances) of the rings were made :—

	Preceding.		Following.		Means.
	Appt.	Dist. 9·5389.	Appt.	Dist. 9·5389.	Dist. 9·5389.
Width of Crape ring	$2''{\cdot}88$ (8)	$2''{\cdot}44$	$2''{\cdot}79$ (8)	$2''{\cdot}37$	$2''{\cdot}41$
From limb to Crape ring . . .	1·76 (9)	1·49	1·54 (8)	1·30	1·40
From limb to edge of bright ring .	4·50 (10)	3·80	4·46 (8)	3·79	3·80

Dec. 31. Seeing good. With 700 diameters the ball, south, essentially touched the edge of the ring, but did not protrude above it. It may have been $\frac{1}{4}''$ less than the edge of the ring. There was not contrast enough to tell. They seemed to coincide. Could see nothing of the ball below the ring, north. It was only steady for moments.

1913 Jan. 18. The best I could make of it was that the ball, south, was almost exactly coincident with the minor axis of the

ring. Possibly it was the slightest bit less, but not exceeding 0″·1. I could see no trace of the ball north of the ring. Could see the polar cap and belts about as in the photograph of 1911 November 19. The Crape ring, where it crossed the ball, was perfectly uniform on both edges. There was no irregularity on it. The ball was well seen through it. At the ansæ it was bright and well defined at its edges, and extended perhaps very slightly more than half way to the ball.

Jan. 25. Seeing very bad with high wind. Could not see the ball on the north side of the rings in moments of steadiness. The south limb seemed to coincide with the rings, but the seeing was too poor to make much out of it.

Jan. 28. With 700 diameters. The south limb was either perfectly coincident with the edge of the ring, or smaller by less than 0″·1. No trace of it protruding from the north edge of the ring. The trace of the Crape ring across the ball was perfectly uniform—no irregularities. Could trace the ball through it up to the bright ring. At the ansæ it was very clear and distinct, a light grey in colour, and very definite. On the preceding side it extended, by estimation, 0·55 of the space to the ball; on the following side, 0·50. The outer ring, at the ansæ, was slightly darker than the inner bright ring. The bright ring near the Cassini division was bright, but shaded off to the Crape ring—that part being the same depth of shade as the outer ring. The line of demarcation between it and the Crape ring was sharp. The rings were a little brighter than the ball. The following measures were made:—

	Preceding.		Following.		Means.
	Appt.	Dist. 9·5389.	Appt.	Dist. 9·5389.	Dist. 9·5389.
	″	″	″	″	″
Width of the Crape ring . . .	2·48 (8)	2·27	2·52 (8)	2·30	2·28
From limb to Crape ring . . .	1·68 (8)	1·53	1·70 (8)	1·55	1·54

The means of the values on this date and on November 17 are, at the mean distance (9·5389):—

	″
Width of Crape ring . . .	2·34
From limb to Crape ring . .	1·47

This makes the Crape ring extend 0·63 the distance from the bright ring to the ball, which is in fairly close agreement with the estimations given in this paper.

The polar cap was dark and well defined.

Feb. 1. The ball was entirely hidden by the ring. It seemed coincident with the south edge, however, but the seeing was not good enough to tell. It may have been slightly less than the ring.

Feb. 8. Measures of the ring made on this date give:

	Appt.	Dist. 9·5389.	Means. Dist. 9·5389.
	″	″	″
From inner bright ring to preceding limb	3·96 (8)	3·70	3·83
From inner bright ring to following limb	4·16 (8)	3·89	4·02

March 4. The polar diameter was a little less than the minor axis of the ring. Could not say how much, as the seeing was poor. With 700 diameters.

Correction.

I have lately detected an error in one of the quantities in my measures of the Saturnian system, printed in *Monthly Notices*, vol. liv. On p. 168 the mean of the values for the outer diameter of the ring is given as 40″·013. This should have been 40″·133.

The final value on p. 171, 40″·108 should, therefore, be changed to 40″·186.

The corresponding diameter is 172,630 miles.

Yerkes Observatory,
Williams Bay, Wisconsin:
1913 *April* 3.

Preliminary Results of Observations made with the Cookson Floating Zenith-Telescope at the Royal Observatory, Greenwich. By A. S. Eddington, M.A., M.Sc.

(*Communicated by the Astronomer Royal.*)

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1. The floating zenith-telescope was designed by the late Mr. Bryan Cookson (*M.N.*, lxi. p. 315), and after his death was presented to the Cambridge Observatory by Mrs. Cookson. It has now been lent to the Royal Observatory, Greenwich, for a period of seven years. In principle it is similar to an ordinary zenith-telescope, the distinctive feature being that the rotation about a truly vertical axis is secured by carrying the Y's of the telescope on an iron annulus, which floats in an annular trough of mercury; thus the levels, which form an important part of the ordinary instrument, are dispensed with. The object-glass has an aperture of

$6\frac{1}{2}$ inches, and a focal length of 65·4 inches. Observations are made photographically. On moderately clear nights the trails of stars from $6^m{\cdot}5$ to $7^m{\cdot}0$ (depending on the Declination) are shown.

Mr. Cookson had made trial of the instrument in a research on the aberration-constant and variation of latitude from 1905 July to 1907 July. The observations have been edited by two of his colleagues (*Memoirs R.A.S.*, vol. lx., pt. ii.). These first results were not considered to be satisfactory, showing at certain periods large deviations from the accepted values of the latitude-variation, and, in particular, the large correction $-0''{\cdot}075$ to the aberration-constant in each of two years. In so delicate a research it was not surprising that the pioneer work revealed difficulties, and it was reasonable to expect that with further experience an improvement would have been made. In planning the new series of observations, we have greatly benefited by having Mr. Cookson's experience as a guide.

For his research Mr. Cookson had used the telescope for observing stars transiting close to the zenith, a reversal being made on each star; but he had designed the telescope for observing Talcott pairs of stars as an alternative, and this latter method has now been adopted at Greenwich. In the Talcott method a pair of stars, one north and the other south at nearly the same Z.D., are observed, the instrument being reversed between the two observations. In either case the quantity measured is the distance between two star-trails. In the first method, in which the trails belong to the same star, this distance is twice the star's Z.D.; in the Talcott method, it is twice the mean Z.D. (taken algebraically) of the two stars. From the changes in the Z.D. the latitude-variation and the aberration-constant can be deduced.

2. The Talcott method appears to possess some important advantages:—

(1) The star-trails, whose distance apart is to be measured, are close together. Thus in the programme now under observation, out of 68 pairs

19 are within 5′ separation,
25 are from 5′ to 10′ apart,
19 are from 10′ to 15′,
5 are over 15′.

This is a great reduction as compared with the other mode of using the instrument. Thus in Mr. Cookson's programme the distances between the pairs of trails ranged up to 3°, the average being about 70′ ($3\frac{1}{2}$ cms.). With the trails close together the réseau can be dispensed with, and the distances measured by a simple comparison with a scale.

(2) The trails are photographed on the meridian, and are thus in the centre of the field. The effect of all errors of adjustment is a minimum here, and the reductions are especially simple.

(3) The trails are parallel to one another (being on the meridian); it is thus possible to make the measures by setting the trail between

a pair of parallel wires in the microscope. In the other method, it is necessary to confine the measures to a number of individual points on the trail—a much less satisfactory proceeding.

Against the method it must be allowed that the observing is slower (not much more than half as fast) than the method of reversal on zenith-stars.

It is a matter of opinion whether the continual alteration of the Z.D. setting of the telescope is an advantage or a disadvantage of the method.

3. *The Observing Programme.*—The conditions for the best determination of latitude-variation and of the aberration-constant are not altogether reconcilable. It was thought best to regard the aberration as the main object of the Greenwich observations, latitude-variation being particularly well cared for elsewhere, and the programme has been devised accordingly. The series should, however, give a quite adequate determination of latitude-variation; and, in particular, there are certain points of doubt connected with the annual z term, on which its evidence may be of interest.

The programme consists of 16 Groups, each involving about an hour of observation and containing 4 or 5 pairs of stars. Each Group is photographed on a separate plate. Table I. shows the limits of each Group and the mean zenith-distance (algebraical) of the stars included in it; it is arranged that this shall be small, in order to eliminate wholly or mainly in the mean of each Group the effects of any change of scale through temperature or other causes. Owing to precession the mean Z.D. changes a little from year to year.

TABLE I.

Group.	No. of Pairs.	Approx. Limits of R.A.	Mean Z.D. North 1913·0.
		h h	′
1	4	0 – 1	−0·8
2	4	2½ – 3½	−0·7
3	4	3½ – 4½	−0·6
4	4	7½ – 8½	−1·1
5	4	8½ – 9½	+0 1
6	4	11 – 12	−0·2
7	4	12 – 13	+0·6
8	4	13½ – 14½	+0·4
9	5	14½ – 16	+0·4
10	4	16 – 17	+1·5
11	4	17 – 18	−0·2
12	5	18 – 19	−0·7
13	5	19 – 20	+1·0
14	4	20 – 21	−0·4
15	5	21 – 22	−1·0
16	4	23 – 24	−0·9

The individual stars range from 0° up to 15° and occasionally 20° Z.D.

The essentials for determining the aberration are that observations should be made as near 6 p.m. and 6 a.m. as practicable, the aberration in Z.D. taking its extreme values at about those times; also the Groups must be observed cyclically in order to eliminate the star-places. The short summer nights in these latitudes are unfavourable for devising a symmetrical programme. The Groups are observed as follows:—

Date.	Evening. Groups	Morning. Groups
Dec. 25 to Jan. 31	2, 3	6, 7
Feb. 14 to Mar. 23	4, 5	8, 9
Mar. 24 to Apr. 26	6, 7	9, 10
Apr. 27 to May 25	8, 9	10, 11
May 30 to June 21	10, 11, 12	
June 22 to July 9	11, 12, 13	
July 14 to Aug. 7	12, 13	14, 15
Aug. 12 to Sept. 19	13, 14	16, 1
Sept. 20 to Oct. 25	14, 15	2, 3
Nov. 11 to Dec. 22	16, 1	4, 5

It will be seen that the scheme during the winter months consists of two interlacing Küstner polygons, which coalesce during the short summer nights. In order to make all the links of equal strength, it is necessary to have specially numerous observations of the summer Groups, since they serve double duty, being required to close both polygons. Happily, the weather conditions usually tend to bring about this distribution of weight automatically.

All the stars are photographed on the meridian; the usual exposure is one minute, but the most northerly stars are given rather longer to obtain a sufficient length of trail. The telescope settings are so arranged that the trails are all contained within 25′ of the centre of the field and along the middle line of the plate. The effects of collimation, level and azimuth error, tilt of plate and of the object-glass are thus reduced to a minimum (it may be recalled that in Mr. Cookson's investigation the tilt of the plate introduced some trouble). In the present arrangement the collimation, level and azimuth errors affect the results so little that the trails would have to be conspicuously askew before an appreciable error could be introduced into the measures.

The meridian is marked on the plate by the shadow of a thick wire in the telescope aperture, against which the plate is placed.

4. *Measurement of the Plates.*—For measuring the plates one of the Greenwich micrometers has been reconstructed with new microscopes. Two microscopes, whose distance apart can be adjusted, are connected by a bar, which moves longitudinally on a horizontal slide. The plate is viewed by one microscope (A) and a glass millimetre-scale is placed under the other (B). Keeping the distance between the two microscopes constant, we can project any length on the plate on to the scale. Each microscope is provided with a micrometer-screw, but the one by which the plate is viewed

is used only for a few hundredths of a revolution, as its "runs" are not easily determined. The measures are made as follows:—A trail having been brought approximately under a pair of close parallel wires in microscope A, settings are made with the micrometer on four different parts of the trail, four very slightly different readings resulting. In microscope B there are two pairs of parallel wires roughly a scale-division apart, and these are set successively on the two nearest scale-divisions. We have thus four readings on the trail and two on the scale—a suitable proportion, as the latter is much the easier observation. The companion trail is then brought under microscope A and the settings made as before.

The curvature of the trails is small but appreciable. As it is constant for each star, provided the same length of trail is always measured, it is unnecessary to apply any correction, except in imperfect trails, when it has been necessary to deviate from the usual course of measurement.

The measurement of a plate, containing 4 or 5 pairs of trails, in one position takes a quarter of an hour; all the reductions, except the star-corrections, can be performed by the enterer then and there. The two stars of a pair are to be regarded as inseparable, only one set of star-constants and one correction is required for the two stars.

5. *Results of the Observations.*—At the time this discussion was undertaken 146 plates had been completely measured and reduced. Of these, 8 show only 2 star-pairs, and have been given half-weight; the remainder contain generally 4 star-pairs, but occasionally 3 or 5.

The plates were measured in the direct position by the writer, and in the reversed position by Mr. Acton. The mean discordance in their measures (latitude deduced from a single star-pair) ranges in the different groups from $\pm''\cdot10$ to $\pm''\cdot17$. The lower value appears to represent the accidental error of the measures; the higher values are due to systematic differences between the measurers which appear in certain star-pairs. These systematic differences, which not infrequently amount to $0''\cdot25$, have been traced to the difference of magnitude of the two stars forming a pair. It appears that each measurer sets too far to the left (as seen in the microscope) on a faint trail as compared with a bright, and the difference arises from the reversal of the plate and not from a difference of personality. The systematic differences should be eliminated in the mean, and accordingly we may take the accidental discordance as $\pm''\cdot10$; and the latitude derived from a single star-pair will thus be subject to a mean error of measurement of $\pm''\cdot05$. This is insignificant compared with the errors from other sources.

At this early stage in the programme the material is insufficient for establishing an adequate connection between the star-places in the different Groups. Consequently, an independent determination of latitude-variation at present is not of much weight. (It will be understood that in a seven-year series of observations the connection between the star-places is derived from the whole series of observations, so that the definitive latitude-variation for the first year is not derived until the end of the period. Fairly approximate results

should, however, be obtainable after two or three years.) In order to obtain preliminary information as to how the work was turning out, the residuals for each plate have been corrected for the latitude variation (provisional results), published by Professor Albrecht from the observations of the International Breitendienst. The mean of the residuals for any Group then represents the error of our adopted star-places for that Group and can be taken out. The final residuals thus corrected will be the errors of our determinations, on the assumption that the International results are correct, and that no correction to the aberration-constant is required.

I find that for the 142 plates (*i.e.* 138 + 8 half-weight) the sum of the residuals irrespective of sign is 11″·08. This must be divided by 127, not 142, because these are discordances from 15 Group-means.* The mean error is thus ±0″·0873, and probable error of latitude deduced from a single plate

$$= \pm 0''\cdot 074.$$

The probable error is much the same in the different Groups. This is a fairly satisfactory result, so far as it goes, though it is decidedly larger than we should deduce from the discordances *inter se* of the individual stars on the plates. If the results of Mr. Cookson's investigation with this telescope are subjected to the same test of comparison with the International determination, the probable error of the latitude derived from a plate is found to be ±0″·120.

6. *The Aberration-Constant.*—Rather out of curiosity than because the result at this stage can have any real value, I have made a determination of the correction to the aberration-constant from the plates now measured.

If z_n is the error of the adopted star-places of Group n,
ϵ the probable error of a plate ($= \pm''\,074$),
$20''\cdot 47\ (1 + \delta\kappa)$ the constant of aberration,
the two polygons give respectively

$$z_{12} + z_{13} - z_{10} - z_{11} - 142\delta\kappa = -''\cdot 255, \text{ probable error } \epsilon\sqrt{5\cdot 96}$$
$$z_{12} + z_{13} - z_{10} - z_{11} - 149\delta\kappa = -''\cdot 015, \text{ probable error } \epsilon\sqrt{8\cdot 15}$$

the difference on the right-hand side being not greater than would be expected from the probable errors.

Combining these with weights 3 : 2, as indicated by the theoretical errors,† we have

$$z_{12} + z_{13} - z_{10} - z_{11} - 146\delta\kappa = -''\cdot 159, \text{ probable error } \epsilon\sqrt{4\cdot 17}.$$

The closing link of both polygons is

$$z_{10} + z_{11} - z_{12} - z_{15} - 16\delta\kappa = -''\cdot 017, \text{ probable error } \epsilon\sqrt{1\cdot 15}.$$

Hence by addition

$$-162\delta\kappa = -''\cdot 176 \pm \epsilon\sqrt{5\cdot 32}.$$

* No results from Group 5 have been used.

† Owing to the polygons beginning to coalesce before the closing side is reached, a small portion of the errors, viz. $\epsilon\sqrt{1\cdot 5}$, is common to both polygons. This leaves the independent errors $\epsilon\sqrt{4\cdot 46}$ and $\epsilon\sqrt{6\cdot 65}$ respectively, giving the weights 3 : 2. The error of the combination is then $\epsilon\sqrt{(1\cdot 5 + \frac{3}{5} \times 4\cdot 46)} = \epsilon\sqrt{4\cdot 17}$. The comparative weakness of the second polygon is due to the absence of results from Group 5.

The correction to the constant of aberration

$$= 20\cdot47\delta\kappa$$
$$= +''\cdot022 \pm ''\cdot021,$$

using the value $''\cdot074$ for ϵ.

7. *Latitude-Variation.* As already stated, this cannot properly be determined until sufficient material has been collected to make a satisfactory connection of the Groups; however, it may be of interest to exhibit the present state of the results. The provisional star-places used depended on the International latitude-variation (but these were, of course, entirely eliminated in the above determination of the aberration-constant). The following corrections to the Group places result from the first year's observations:—

$z_1 = -''\cdot013$	$z_7 = -''\cdot007$	$z_{12} = +''\cdot062$
$z_2 = -\cdot063$	$z_8 = -\cdot006$	$z_{13} = +\cdot059$
$z_3 = -\cdot038$	$z_9 = -\cdot006$	$z_{14} = -\cdot002$
$z_4 = +\cdot004$	$z_{10} = +\cdot030$	$z_{15} = -\cdot036$
$z_6 = -\cdot074$	$z_{11} = +\cdot068$	$z_{16} = +\cdot027$

Using these and the corrected value of the aberration-constant, the following comparison with the International Breitendienst is obtained:—

Middle Date.	Number of Plates.	Latitude-Variation. Cookson *minus* International.
1911 Sept. 10	11	$-''\cdot05$
Oct. 7	$14\frac{1}{2}$	$+\cdot02$
Dec. 1	$12\frac{1}{2}$	$+\cdot05$
1912 Jan. 10	$9\frac{1}{2}$	$+\cdot07$
Mar. 3	6	$-\cdot06$
Apr. 10	16	$+\cdot02$
May 10	12	$\cdot00$
June 10	$12\frac{1}{2}$	$-\cdot10$
July 1	10	$-\cdot07$
July 25	$14\frac{1}{2}$	$+\cdot01$
Aug. 30	$10\frac{1}{2}$	$+\cdot01$
Oct. 7	7	$+\cdot08$
Dec. 1	6	$-\cdot01$

8. An interesting insight into the sources of error of the instrument is obtained by arranging the residuals * according to the date of the observations. This is done in Table III. There is strong evidence of something which may be called "the error of the night," an error which is repeated on consecutive plates. The best examples are:

1912 May 19.	Residuals of	$-''\cdot10, -''\cdot09, -''\cdot02, -''\cdot11$
24.	,,	$+\cdot08, +\cdot09, +\cdot01, +\cdot20$
July 14.	,,	$+\cdot18, +\cdot19, +\cdot09, +\cdot07$

These, as it happens, are the *only* instances in which four strictly consecutive plates have been taken, because as a rule the evening and morning watches are separated by an interval of some hours.

* *I.e.* the residuals discussed at the end of § 5.

Other rather striking cases of "error of the night" are 1911 Sept. 18, Oct. 12, Dec. 19, 1912 Apr. 24 (morning), June 18, Oct. 2; the only exception which may fairly be set against these is 1911 Oct. 11 (residuals $-''\cdot11$ and $+''\cdot13$).

The evidence may be given statistically. The mean difference of the residuals of two consecutive plates is $\pm''\cdot073$, which would give the probable error of a single plate as $\pm''\cdot045$. We have previously found it to be $\pm''\cdot074$; and we must infer that the large excess is due to some source of error which affects consecutive plates equally. It seems permissible to analyse the errors as follows:—

Error of measurement . . .	$\pm''\cdot022$
Error of the star-images . . .	$\pm''\cdot039$
Error of the night	$\pm''\cdot059$
Resultant	$\pm''\cdot074$

It is clearly of great importance to endeavour to trace the cause of the error of the night, not only because it is the largest contributor to the error, but because it remains to be seen whether it is purely accidental or can affect the aberration-constant systematically.

TABLE III.

Residuals of Individual Plates (*corrected for latitude-variation*) (*unit* = 0″·01).

Group		13	14	15	16	1	2	3	4
1911 Sept.	7	−19	− 5						
	8	−12			0	+ 4			
	15	+13	+ 6						
	16				+ 4	− 4			
	18				−16	−12			
	21						+ 4*	− 8	
	25		−20*						
Oct.	3							+ 5	
	4		0	+ 6					
	6		− 7	− 5					
	9		− 1						
	10		+10						
	11						−11	+13	
	12		−12	−12					
	18		0*	+ 4					
Nov.	15				− 1				
Dec.	5				(+32)†	− 2			
	6				− 9	− 8			
	7				+18*				
	8								− 3
	11					+ 8			+12
	12				+12	+ 3			
	13					+ 6			
	19				+18	+10			

* Two pairs of trails only, – half-weight.

† The two plates on December 5 were accidentally exposed film downwards. The plate for Group 1 does not seem to have suffered.

TABLE III.—*continued.*

Group		2	3	4	6	7	8	9	10
1912 Jan.	5	+ 1	+ 2						
	27				+10	+ 5			
	28	− 4*	− 5		+ 3	+ 9			
	29	+ 9	− 4						
Feb.	29			+ 3					
Mar.	6						0	−18	
	8						+ 1	− 3	
	15			−11					
Apr.	1				+12	− 5		+ 9	+11
	2							+ 4	+ 6
	3				−16	− 3			
	10					− 1			
	16				− 8				
	18							− 8	−24
	24				0	− 3		+17	+24

Group		8	9	10	11	12	13	14	15
1912 Apr.	29				+13				
May	13		− 3	+14					
	17			+ 4					
	19	−10	− 9	− 2	−11				
	24	+ 8	+ 9	+ 1	+20				
	31			−11	+ 9	+ 6			
June	3			− 7	+ 5	− 4			
	8			− 7	+10*				
	18				−23	−22			
	20			− 4	− 6	− 2			
	23					+ 2	+ 3		
	26				− 7	− 4	−10		
	28				−11	+ 2	0		
July	3				+11				
	11					+ 2		− 4	
	12						−11		
	13						+11	− 5*	
	14					+18	+19	+ 9	+ 7
	15							− 1	+ 7
	17					− 8			
Aug.	1							− 6	−14
	2					+ 4	+12		

Group		13	14	15	16	1
Sept.	2		+12		+12	+ 2
	6	+15			+ 1	+ 1*
	9	−11	+ 3		−14	
	20	+ 8	− 6			
	23			− 2		
Oct.	2		+13	+17		
	7		− 3	− 3		
	9		+ 8	− 2		
Dec.	4					− 7
	16				+ 5	+ 2
	18				− 8	− 3
	20					− 3

I think there is some difference between the "error of the night" shown in these observations and the discordances found in the previous research at Cambridge. In that work the chief difficulty seems to have been that at certain times the instrument showed a discordance, which remained systematic for several nights or even weeks in succession. The discordances too were considerably larger on the average than those now found. It seems that they must have been of a different nature from the discordances found in the present investigation, which persist only for a few hours, and show little or no systematic tendency when grouped week by week. I would suggest that the former discordances arose from the difficulty of measuring the long distances between the trails; possibly a slight difference in printing the réseau on successive batches of plates would cause such deviations. Mr. Cookson's programme did not involve taking two plates consecutively, so that there is no material for a discussion of the kind we have just considered.

9. As regards the cause of the present discordances, it seems to me unlikely that they are in any way due to a changing error of flotation of the instrument. A temperature-gradient across the mercury-trough could hardly persist unchanged for four hours during a summer night in a room with closed doors, the mercury being often disturbed by reversals of the instrument. Bubbles under the float would be more likely to produce a discordance lasting for a number of days, than one which is limited to a single evening, and which changes in the interval while the instrument is untouched. If the flotation is correct, the only other possible source of error is, I think, in the refraction. It may here be remarked that the accordance between the first and second plates of an evening's observation shows that no error has arisen through taking observations before equilibrium between the atmosphere outside and inside the hut has been established after the opening of the roof.

The most likely meteorological condition with which the residuals might be correlated seemed to be direction of the wind. I have compared this for each night of observation. As a sample I take the "remarkable" dates mentioned above.

Date.	Residuals.	Wind.
1912 May 19	$-''\cdot10$, $-''\cdot09$, $-''\cdot02$, $-''\cdot11$	SW, calm.
24	$+''\cdot08$, $+''\cdot09$, $+''\cdot01$, $+''\cdot20$	NE, NNE.
July 14	$+''\cdot18$, $+''\cdot19$, $+''\cdot09$, $+''\cdot07$	E, calm.
1911 Sept. 18	$-''\cdot16$, $-''\cdot12$	WSW.
Oct. 12	$-''\cdot12$, $-''\cdot12$	SE.
Dec. 19	$+''\cdot18$, $+''\cdot10$	SSW, S.
1912 Apr. 24 (morn.)	$+''\cdot17$, $+''\cdot24$	NE.
Jun. 18	$-''\cdot23$, $-''\cdot22$	WSW, calm.
Oct. 2	$+''\cdot13$, $+''\cdot17$	NE.

The evidence here seems to point to a connection of NE. winds with positive residuals, and SW. with negative residuals; Dec. 19, however, forms a marked exception. The other dates support this on the whole, though exceptions are to be found. I find there is no indication of the strength of the wind being important (it was not very strong on any of these dates, the maximum being 14 miles per hour on Oct. 2), nor is there any explanation why the effect was unusually strong on the particular dates mentioned.

10. Let us consider how anomalous refraction may arise. It will be sufficient to consider a zenithal ray. The actual rays, it is true, are distributed symmetrically within about 15° on either side of it; but it is clear from the results for individual stars that, when the plate-residual is large, all the star-pairs contribute to it.

If θ is the inclination of the surface of equal density of the air to the horizontal, i the inclination of a ray to the vertical, the differential equation of the path is

$$di = \frac{d\mu}{\mu} \tan (i + \theta).$$

For a zenithal ray i may be put zero on the right-hand side and

$$di = \tan \theta \frac{d\mu}{\mu}.$$

To find what can happen within 50 feet of the ground (*i.e.* the disturbance due to surrounding buildings), we have

$$\int \frac{d\mu}{\mu} \text{ for 50 feet} = \frac{1}{600} \times 57''$$

the diminution of pressure being $\frac{1}{600}$ of an atmosphere. (For adiabatic equilibrium, the integral would be rather smaller.)

Thus to produce a deflection of 0″·1 within 50 feet of the ground the average inclination of the surfaces of equal density must be 45°. This is perhaps quite possible. We may assume that the equal-pressure surfaces are nearly horizontal, otherwise there must be strong winds, and the deflections, as we have seen, occur on comparatively calm nights. But a fall of temperature as we move horizontally from the Observatory at the rate of 1° C. per 100 feet would tilt the equal-density surfaces through 45°. A very local gradient (*e.g.* a total fall of 0°·2 C. in 20 feet) would be sufficient, provided it extended up to 50 feet over the Cookson telescope.

It seems then not unreasonable to suppose that a local distribution of temperature, due to the proximity of the Observatory buildings, might produce refractions of the necessary magnitude. It is something in favour of this hypothesis that, if the courtyard contained warmer air than that outside, the refraction would be in the direction to give negative residuals, which we have found to be associated with south-westerly winds. Now the courtyard is sheltered on the south and especially the west, but exposed to the north and east, so that south and west winds are most favourable

to the continuance of such a temperature gradient, whereas north and east winds would stir the air thoroughly.

If, instead of confining ourselves to the atmosphere near the ground, we consider a gradual deflection of the zenithal ray through the whole atmosphere, the necessary gradients are much smaller. Since the normal refraction is $0''{\cdot}1$ at a zenith distance $0^\circ{\cdot}1$, an average tilt of $0^\circ{\cdot}1$ of the surfaces of equal density throughout the atmosphere would produce a deflection of $0''{\cdot}1$ in the zenithal ray. This tilt amounts to 10 feet per mile, and would correspond to a barometric gradient of 1 inch per 100 miles, or a temperature gradient of 1° C. in ten miles. The barometric explanation is, of course, impossible, and the thermometric one seems not much better. It should be remembered that these gradients must be supposed to persist for three or four hours.

Intermediate between these two extremes, we might consider the effect of the configuration of Greenwich hill, the park, and the river. This presumably would affect the slope of the surfaces of equal density up to a height of at least several hundred feet. This possibility offers much scope for speculative explanations, but I do not think we have sufficiently definite knowledge to make the discussion profitable.

Whether we consider that the deflection occurs in the ground layers, in the first thousand feet, or even in the whole atmosphere, it seems certain that the cause must be a temperature rather than a barometric gradient. Also the fact that the large residuals are associated with feeble winds seems significant, for a stirring up of the atmosphere must tend to reduce the horizontal gradients and regularise the refraction.

A Photographic Determination of the Proper Motion of 250 *Stars in the Neighbourhood of* Σ 443.

By Dr. A. A. Rambaut, F.R.S., Radcliffe Observer.

During the years 1904 to 1908 the principal work to which the 24-inch photo-refractor of the Radcliffe Observatory was devoted consisted in the photographic determination of the relative parallaxes of stars down to the 11th-12th magnitudes in certain selected regions of the sky. A résumé of the progress of this investigation has been published in the Report of the Council for each year. Here it is only necessary to remark that in the exposure of the plates and the methods of measurement and reduction Professor Kapteyn's now well-known scheme has been closely followed.

In the course of this work, as will be easily understood, it was found impossible, owing to unfavourable weather, to expose all the plates exactly according to the programme arranged at the inception of the work. Thus, for instance, in the case of five plates of the region of Nova Persei, exposed for the first time in

the early part of 1905, bad weather during the autumn entirely prevented their being exposed at the next season of maximum parallax. This difficulty was, however, surmounted by allowing them to remain over for another year, when they were successfully exposed in the autumn of 1906, the final third season exposures being delayed until the spring of 1908. In this way the intervals between the first and second exposures and between the third and fourth were each about a year and a half instead of half a year, and the deduced parallax comes out with the same weight as if the plates had been exposed in the normal way.

A similar difficulty occurred in the case of four plates of the Hyades group, and was got over in the same way.

With one plate, however, representing the region surrounding the star Σ 443, we were not equally successful. It was first exposed in January 1905, but, owing to unfavourable weather, the region was missed in the autumn of 1905 and 1906 at the time of large parallax-factor. In the year 1907 we were bringing the parallax investigation to a conclusion, and making arrangements to commence the determination of the proper motion of faint stars down to the 14th magnitude, in accordance with Professor Kapteyn's "Plan of Selected Areas." This plate (37B) was, accordingly, not again exposed for parallax, but was kept in its tin box, wrapped closely in paper, for eight years, as a test of the keeping properties of the plates under such conditions.

With regard to this point it is satisfactory to note that the plate seems to have suffered very little in the interval of eight years. There is a sort of thin, mottled veil of fog, which seems to be chiefly due to the effect of the paper envelope which contained the plate inside the tin box, but this is so slight that it cannot have any effect on the ease or accuracy of measurement. The three images corresponding to the earlier exposure are rather more diffuse than the later ones. This difference may be due either to a deterioration of the latent images in the film during the period of storage, or to a difference in the atmospheric conditions at the first and second exposures, or again, to the fact that in the interval between the exposures the objective of the telescope had been readjusted with regard to the centring and separation of the lenses; but it is impossible to discriminate between these three causes, or to say how much of the effect is attributable to one cause and how much to another. The measurer of the plate has entered the following remark in his note-book: "The images on this plate are very poor as compared with the parallax plates. The two seasons do not differ very greatly in definition, but the second season images are somewhat darker."

The remarks of the observers at the telescope are as follows:—

Earlier exposure, 1905. *Observers A.A.R. and C.* (Dr. Rambaut and Mr. McClellan).—"This plate should be duplicated if possible. The instrument was shaken by the wind during the second and third exposures. Bright moonlight; some haze at times."

Later exposure, 1913. *Observers H. B. and B.* (Mr. Barrett and

Mr. Balk).—"Somewhat unsteady at times in wind. Definition poor during part of first exposure."

The results of the measures of this plate are exhibited in Table II., below. Here it will be convenient to give some particulars with regard to the exposure and measurement of the plate and the mode of reduction.

The *earlier* exposures were made on 1905, Jan. 14, at the following sidereal times:—

	h	m	s		h	m	s
(1) From	3	46	47	to	3	56	47
(2) ,,	3	58	50	,,	4	8	50
(3) ,,	4	11	42	,,	4	21	42
Mean	3	59	6	to	4	9	6

The *later* exposures were made on 1913, Feb. 4, as follows:—

	h	m	s		h	m	s
(1) From	4	50	53	to	5	0	53
(2) ,,	5	3	46	,,	5	13	46
(3) ,,	5	14	35	,,	5	24	35
Mean	5	3	5	to	5	13	5

Each star on the plate is represented by two rows of images arranged thus ⁝⁝, the displacements in declination being approximately equivalent to 30″, those in R.A. to 25″. The differences of the co-ordinates of corresponding images were measured in two directions at right angles to each other, corresponding to the directions of standard co-ordinates for the centre of the plate. The differences of the x co-ordinates were first measured, two pointings being made on each image. The plate was then turned through 180° and similar measures effected. After this the plate was turned first through one right angle and then through three right angles, and measures of the differences of the y co-ordinates made in an exactly similar way. The measures were arranged in this manner so that both series might be made with the same micrometer-screw.

Now if, for any star, ν represents the resulting mean difference in either co-ordinate, μ the annual proper motion in that direction, t the interval of time between the first and last exposures, π the annual parallax, h_1 and h_2 the parallax factors corresponding to the dates of exposure, and K a correction including the effects of refraction, aberration, change of scale, tilt of plate, and the intentional displacement of the images, we have the relation

$$\nu = \mathrm{K} + (h_2 - h_1)\,\pi + t\mu.$$

We do not know the parallaxes of all the stars on the plate, but the discussion of another plate (34B), which contains about three-quarters of the stars now under consideration, shows that the parallax of Σ 443 is not greater than 0″·1, whilst the other parallaxes are all smaller than this. The values of h_2 and h_1, corresponding to Feb. 4 and Jan. 14, are so nearly equal (being −0·93 and −0·80

respectively) that the effect of the parallax on the deduced proper motion is quite insensible, and has therefore been neglected.

The terms of the second order in the refraction and aberration being found to be in all cases quite negligible, I have taken a linear expression to represent K, so that each measure affords an equation of condition of the form

$$t\mu = ax + by + c + \nu.$$

If n be the number of stars, we have therefore n such equations from which to deduce $n+3$ unknown quantities, namely, the n separate values of μ and the three constants a, b, and c. In order to obtain the three additional equations required for the solution we impose the condition that the values of a, b, and c shall be such as will make $\Sigma(\mu^2)$ a minimum. As is well known, this is equivalent to solving for a, b, and c, by least squares from the n equations.

$$\left.\begin{aligned} ax_1 + by_1 + c + \nu_1 &= 0 \\ ax_2 + by_2 + c + \nu_2 &= 0 \\ \cdots\cdots\cdots\cdots\cdots & \end{aligned}\right\} \quad . \quad . \quad . \quad (1)$$

The residuals so found, each divided by t, give us, then, the values of μ_1, μ_2, etc.

We cannot of course rigorously deduce the probable error of a proper motion thus obtained, but, as Kapteyn has done for the parallaxes, we may obtain an upper limit for the probable error by attributing the residuals of the equations (1) to errors of observation alone. In this way we find the expression

$$\rho_3 = 0{\cdot}6745\sqrt{\frac{[\mu\,\mu]}{n-3}}$$

as an upper limit to the probable error of a proper motion deduced from three pairs of images.

In the case before us we have, in the solution of equations (1), excluded the measures of Σ443, which was known to have a large proper motion, and 47 other stars whose images were noted as being too faint, defective, or for any reason unsuitable for accurate measurement.

Thus we obtain from measures of 201 stars,

	R.A.	Decl.
$\rho_3 =$	$\pm 0''{\cdot}0111$	$\pm 0''{\cdot}0106.$

These results are interesting both for their own sake and as affording an indication of the degree of accuracy which may be expected from the plates we are at present engaged in taking for Kapteyn's "Selected Areas."

In the present case we had three pairs of images of each star, whereas in the case of the plates which we are now storing only two exposures are made at each epoch. In order therefore to estimate the probable error of a proper motion deduced from one of these plates, the values of ρ_3 given above ought to be multiplied by the factor $\sqrt{\frac{3}{2}}$, or 1·225. On the other hand, it must be remarked that the interval between the two epochs of exposure in

the case of this plate was only 8·06 years, whereas it is intended to make the interval for the plates of the "Selected Areas" ten years at least. For this reason we should have to multiply ρ_3 by 0·8, or divide by 1·25.

Thus we may take the above values of ρ_3 as indicating very closely the maximum value of the probable error of the proper motion as deduced from one of our plates. Or about one-hundredth of a second of arc in either co-ordinate. This is, however, on the assumption that the plates are of the same quality as 37B, now under discussion. But, as has been seen above, this plate was exposed under rather unfavourable circumstances at both epochs, the telescope being on each occasion more or less disturbed by wind, and the images, even those corresponding to the exposure of 1913, are decidedly inferior to those we usually obtain.

We may therefore, I think, conclude that for a plate taken under normal conditions in accordance with our present programme, the probable error of a proper motion in either co-ordinate will prove to be something less than $\pm 0''\cdot01$—a decidedly satisfactory result.

With regard to Table II. below, it should be remarked that the stars are arranged in order of the y co-ordinate, which is very nearly the same as that of decreasing declination. The limits of the field of the photograph are:

	h	m	s		h	m	s	
In R.A.	3	35	30	and	3	42	0	} 1900·0.
,, Decl.	+40°	30′		,,	+41°	45′		

The magnitudes given in the second column are approximate only, and are obtained by comparison with the stars of the B.D., as indicated in the footnotes, reduced to the Harvard scale. Beyond $11^m\cdot4$ the magnitudes depend on a somewhat uncertain extrapolation.

The third and fourth columns of the table contain the approximate R.As. and Declinations of the individual stars; while the two last columns exhibit the resulting proper motions $\mu_\alpha \cos\delta$ and μ_δ.

To obtain μ_α, the proper motion in R.A., the quantities given in the fifth column must be multiplied by $\frac{1}{15}$ sec. δ.

If we exclude $\Sigma 443$ itself, which is represented in the list below by Nos. 123 and 124, all the quantities contained in these last two columns are very small, amounting at most to about five or six times their probable error. Thus it appears that amongst the stars on this plate there are no other cases of *large* proper motion. If there is any magnitude equation, it is not marked enough to make itself felt within the range of magnitude of the stars examined (viz. 7·5 to 11·8).

The proper motions deduced in this way are obviously of a

differential character, and indicate the movements of the individual stars relatively to the mean of the 201 stars, most of them less than the 11th magnitude, employed in the least squares solution.

The proper motion of Σ443 itself has been several times investigated. The common movement of the two components of the pair was first pointed out by Argelander in the *Bonn Observations*, vol. vii. p. 114, No. 43. In the *Publications of the Cincinnati Observatory*, vol. 12, Professor Porter deduces very similar results. The values published by Professor Küstner in the *Bonn A.G. Catalogue* again agree very closely with the earlier results. More recently the last-named authority has, in the *Veröffentlichungen der Königlichen Sternwarte zu Bonn*, No. 2, published revised values of the proper motion in R.A. and Declination which do not agree quite so closely with those found previously.

These various results are given in the following table:—

TABLE I.

Proper Motion of Σ443.

	R.A.	Decl.	Total P.M.	Position Angle.
	s	"	"	° ′
Argelander (*Bonn*, vii.) . . .	+0·0536	−1·233	1·375	153 49
Porter * (*Cincin.*, 12) . . .	+0·053	−1·24	1·377	154 14
Küstner (*A.G.*)	+0·0534	−1·233	1·373	153 55
Küstner * (*Veröff. Bonn*, No. 2) .	+0·0509	−1·228	1·356	154 53
Mean of the four	+0·0527	−1·233	1·370	154 13
Mean of Radcliffe results . .	+0·0521	−1·229	1·362	154 24

The last line but one contains the mean of the four different determinations; and in the last line is given for comparison the mean of the results for the two components as deduced from the Oxford photographic measures. The close agreement of these two results shows, I think, with what precision it is possible to deduce the proper motion by the photographic method, even though the interval be, as in this case, only slightly over eight years. At the same time an examination of the separate results for Nos. 123 and 124 illustrates a defect of the photographic method from which apparently the visual observations are comparatively free. For it will be seen that there is a difference between the resulting values for the separate components amounting to 0″·06 in R.A. and 0″·04 in Declination. There can hardly be a doubt that a discrepancy of this order, which is quite

* The values of total proper motion and position angle given in Burnham's *General Catalogue of Double Stars*, Part ii. p. 365, are:—

Porter . .	1″·358	in 155°·0
Küstner . .	1″·437	,, 148°·7.

Neither of these agrees with the published values of the proper motion in R.A. and Decl. as exhibited in the first two columns of the above table. In the latter case this is probably due to an error in computation.

irreconcilable with micrometer measures reaching back to 1822, is due to the mutual deformation of the images of the two rather close components. In the case before us the centres of the two images on the plate are separated by less than one-third of a millimetre, and each image appears slightly flattened on the side nearest the other. If this flattening of the images were precisely alike at both epochs, it is possible that it would not affect the resulting proper motion; but if the images corresponding to one epoch were slightly more flattened than those obtained at the other, some effect of this sort must arise.

For this reason I am inclined to regard the difference between the results for the two components of this double star as a spurious effect due to photographic distortion. When we take the mean for the two components, however, this effect ought to disappear. In this way we obtain the results contained in the last line of Table I., which may thus be regarded as the definite proper motion for this binary system as deduced from the photographic plate. Its close agreement with the mean of the results deduced from the visual observations shows, as we should on other grounds have expected, that the mean motion of the 201 stars included in the least squares solution is practically zero, and hence that the absolute proper motions of any one of them is excessively small.

The 11th magnitude star (C) near Σ 443 (Tarrant: *Ast. Nach.*, 2898) has not been measured, but it is clear that it does not share in the common proper motion of A and B. In this connection I may perhaps be permitted to point out a misprint in Lewis's valuable *Measures of the Double Stars of Struve* (*Memoirs of the Royal Astronomical Society*, vol. lvi.). In the last line of page 99 of that volume, *for* 29″·94 *read* 39″·94 as the value of the distance AC.

The symbols : and :: placed against the numbers in the 5th and 6th columns of Table II. indicate slight degrees of uncertainty attaching to the results. The colon shows that (i) one of the six images of the star in question is somewhat defective, but not so much so that the measures need be regarded as unreliable; or (ii) in the case of the brighter stars, that the size of the images may possibly render the results of less weight than those for a star of average brightness. The double colon indicates that (i) there is some decided defect, or that more than one image is slightly defective; or (ii) that the size of a bright star renders the micrometer settings difficult. The results accompanied by a double colon are to be considered as rather uncertain, but not wholly unreliable. Any measures which would fall under the latter description would be rejected.

In conclusion, I desire to acknowledge the aid I have received from Messrs. Robinson and Barrett of this Observatory, in preparing the material for this paper. The measures were made by Mr. Barrett with the "Grubb" measuring machine, and the reduction of the observations was carried out by these two gentlemen in collaboration.

TABLE II.

No.	Approximate Magnitude.	R.A. 1900·0.			Decl. 1900·0.		Annual Proper Motion. $\mu_\alpha \cos\delta$.	μ_δ.
		h	m	s	°	′	″	″
1	10·5	3	40	13	+41	44·4	+0·007	+0·021
2	10·8	3	41	7	+41	44·1	+0·018	+0·024
3	11·4	3	40	34	+41	44·2	−0·003	−0·010
4	10·2	3	38	40	+41	43·9	−0·017	+0·020
5	11·4	3	39	53	+41	43·6	−0·022	0·000
6	10·2	3	41	31	+41	43·2	−0·007	+0·017
7	11·4	3	39	31	+41	43·0	+0·005	+0·035:
8	11·1	3	37	53	+41	42·8	+0·027	−0·015
9	10·5	3	39	39	+41	41·8	−0·013	+0·011
10	11·1	3	37	47	+41	41·8	−0·005	−0·018
11	10·8	3	37	44	+41	41·5	−0·017	−0·002
12	7·5	3	38	46	+41	41·5	−0·009::	+0·005::
13	11·4	3	39	31	+41	41·5	+0·004	+0·048
14	10·2	3	41	56	+41	40·6	+0·018	+0·016
15	11·4	3	39	27	+41	40·7	−0·032::	+0·023
16	11·1	3	36	42	+41	40·4	+0·020	+0·006
17	10·8	3	38	19	+41	40·3	−0·030	−0·015
18	10·2	3	40	51	+41	40·1	+0·007	+0·003
19	11·8	3	38	29	+41	39·9	+0·014	−0·031
20	11·1	3	37	37	+41	39·8	+0·001	+0·001
21	10·5	3	39	7	+41	39·4	−0·007	−0·005
22	11·4	3	39	12	+41	39·1	−0·022	−0·058:
23	10·5	3	37	23	+41	38·4	−0·002	+0·011
24	11·1	3	41	21	+41	37·6	−0·016	0·000
25	10·2	3	39	54	+41	37·1	−0·011	−0·015
26	11·1	3	36	52	+41	36·7	+0·027	+0·012
27	10·8	3	39	42	+41	36·6	−0·006	−0·005
28	10·5	3	40	27	+41	36·6	+0·002	+0·009
29	9·6	3	35	51	+41	35·9	−0·003::	+0·008::
30	11·1	3	36	56	+41	35·4	+0·003	−0·006
31	11·4	3	40	49	+41	34·9	−0·029	+0·021
32	10·2	3	37	36	+41	34·7	+0·004	−0·048
33	9·3	3	36	36	+41	34·1	+0·031	−0·022
34	9·9	3	36	40	+41	33·4	+0·001	−0·008
35	11·4	3	37	13	+41	33·0	+0·019	+0·009

No. 12 B.D. +41° 742
,, 33 ,, ,, 739.

TABLE II.—*continued.*

No.	Approximate Magnitude.	R.A. 1900·0.			Decl. 1900·0.		Annual Proper Motion. $\mu_\alpha \cos\delta$.	μ_δ.
		h	m	s	°	′	″	″
36	11·4	3	39	43	+41	32·7	−0·009	+0·029
37	8·0	3	40	8	+41	32·7	−0·066	+0·066
38	9·6	3	35	53	+41	32·0	+0·002	−0·016
39	11·8	3	37	59	+41	31·9	−0·014:	+0·014
40	10·8	3	38	21	+41	31·6	−0·013	+0·002
41	11·8	3	39	2	+41	31·5	−0·018	+0·012
42	10·8	3	39	43	+41	31·4	−0·001	−0·014
43	11·4	3	40	27	+41	30·8	−0·001	+0·018
44	10·8	3	37	55	+41	30·5	+0·008	+0·020
45	9·9	3	36	55	+41	30·2	+0·014	+0·008
46	11·4	3	37	1	+41	30·1	−0·009	−0·003
47	9·0	3	35	31	+41	29·7	+0·020	−0·048
48	11·4	3	39	24	+41	29·8	+0·010	−0·009
49	11·4	3	39	25	+41	29·8	+0·007	−0·003
50	11·4	3	39	10	+41	29·8	+0·007:	+0·003
51	9·6	3	37	17	+41	29·5	+0·002	+0·003
52	11·4	3	37	46	+41	29·5	−0·007	−0·003
53	9·3	3	39	18	+41	29·3	−0·003	−0·022
54	11·4	3	36	34	+41	29·1	−0·002	−0·012
55	11·1	3	39	4	+41	29·0	+0·007	−0·002
56	11·4	3	38	2	+41	28·8	+0·034	−0·024
57	11·1	3	35	33	+41	28·6	−0·011:	−0·051
58	11·4	3	38	28	+41	28·4	−0·002	+0·017
59	11·4	3	39	40	+41	28·0	+0·014	−0·010
60	11·4	3	38	52	+41	27·6	+0·023	+0·026
61	11·4	3	41	46	+41	27·2	0·000:	+0·030
62	11·4	3	40	32	+41	27·0	0·000	+0·033
63	11·4	3	39	49	+41	27·0	+0·023	−0·019
64	11·1	3	37	56	+41	26·5	+0·005	−0·003
65	11·1	3	37	8	+41	26·5	+0·002	−0·008

No. 37 B.D. +40° 749.
,, 47 ,, ,, 736.
,, 51 ,, ,, 740.
,, 53 ,, ,, 744.

TABLE II.—*continued.*

No.	Approximate Magnitude.	R.A. 1900·0. h	m	s	Decl. 1900·0. °	′	″	Annual Proper Motion. $\mu_\alpha \cos\delta$. ″	μ_δ. ″
66	9·9	3	37	3	+41	26	·3	−0·003	+0·004
67	10·5	3	39	17	+41	25	·3	−0·011	−0·006
68	11·4	3	41	34	+41	25	·2	−0·013:	+0·008
69	11·4	3	36	41	+41	24	·6	+0·007	−0·005
70	11·4	3	36	32	+41	24	·2	−0·001	0·000
71	11·1	3	41	5	+41	24	·2	−0·018	+0·018
72	11·8	3	37	25	+41	24	·1	+0·021:	+0·021
73	11·4	3	40	1	+41	24	·0	−0·007	−0·006
74	10·8	3	39	51	+41	23	·9	−0·009	+0·011
75	9·0	3	40	5	+41	23	·7	+0·003	−0·001
76	11·8	3	37	38	+41	23	·3	+0·012::	+0·009
77	10·8	3	40	5	+41	22	·8	+0·004	+0·004
78	10·2	3	37	37	+41	22	·7	+0·011	−0·001
79	11·4	3	38	53	+41	22	·6	−0·016	−0·007
80	11·4	3	38	8	+41	21	·7	−0·012:	−0·015
81	10·2	3	38	10	+41	21	·5	+0·001	+0·006
82	11·8	3	40	38	+41	20	·9	+0·006:	+0·008
83	11·8	3	40	36	+41	20	·9	+0·008:	−0·015
84	11·4	3	41	30	+41	19	·8	−0·018	−0·004
85	11·4	3	41	13	+41	19	·2	+0·007	−0·014
86	11·8	3	38	5	+41	19	·2	−0·011	+0·015:
87	11·8	3	38	58	+41	18	·7	+0·026	+0·005
88	8·5	3	40	31	+41	17	·1	+0·048	+0·017
89	11·4	3	36	56	+41	16	·6	+0·005	−0·008
90	11·4	3	41	59	+41	15	·7	+0·004	0·000
91	11·4	3	40	23	+41	15	·8	−0·002::	+0·006
92	10·5	3	39	59	+41	15	·2	−0·012	−0·002
93	10·8	3	39	46	+41	15	·0	−0·009	+0·002
94	10·5	3	37	51	+41	14	·8	−0·020	−0·003
95	11·1	3	38	55	+41	14	·6	−0·011	−0·006

No. 75 B.D. +41° 748.
,, 81 ,, ,, 741.
,, 88 ,, ,, 752.

TABLE II.—*continued.*

No.	Approximate Magnitude.	R.A. 1900·0.			Decl. 1900·0.		Annual Proper Motion. $\mu_\alpha \cos\delta$.	μ_δ.
		h	m	s	°	′	″	″
96	10·8	3	40	19	+41	14·6	−0·001	−0·007
97	11·1	3	39	1	+41	14·6	−0·003	+0·015
98	11·8	3	41	21	+41	13·6	+0·008::	+0·021:
99	11·4	3	38	21	+41	13·7	+0·006	−0·018
100	11·1	3	35	40	+41	13·5	−0·007	−0·032
101	10·8	3	39	44	+41	13·6	−0·003	+0·010
102	11·4	3	41	3	+41	13·1	+0·036	−0·010
103	11·8	3	40	13	+41	12·9	−0·016	−0·002
104	11·4	3	40	44	+41	12·7	+0·010	+0·001
105	10·2	3	37	29	+41	12·6	−0·009	+0·008
106	11·4	3	40	5	+41	12·3	+0·016	+0·005
107	11·4	3	39	38	+41	12·2	−0·021	−0·002
108	11·8	3	40	18	+41	12·2	+0·013:	+0·004:
109	11·4	3	36	17	+41	11·3	−0·003	+0·023
110	9·9	3	36	15	+41	11·3	−0·003	+0·008
111	11·8	3	40	13	+41	11·4	+0·011::	−0·026::
112	9·6	3	38	55	+41	11·3	−0·052	−0·037
113	11·4	3	37	23	+41	11·3	−0·012	−0·005
114	11·8	3	40	30	+41	11·1	+0·008:	+0·016:
115	11·1	3	37	31	+41	11·0	−0·004	+0·008
116	11·8	3	40	10	+41	10·8	−0·013	+0·010
117	11·8	3	36	54	+41	10·3	−0·012:	+0·016
118	11·8	3	41	32	+41	10·1	−0·030	−0·004
119	10·2	3	40	44	+41	9·6	−0·011	+0·002
120	11·4	3	39	10	+41	9·6	−0·014	+0·019
121	10·5	3	39	31	+41	9·4	−0·014	−0·014
122	11·4	3	39	1	+41	9·0	+0·002	−0·021
123	9·3	3	40	13	+41	8·9	+0·559	−1·248
124	8·7	3	40	13	+41	8·8	+0·619	−1·209
125	10·2	3	41	50	+41	8·1	+0·014	−0·008

No. 110 B.D. +41° 738.
,, 112 ,, ,, 743.
,, 123–4 ,, ,, 750.
,, 125 ,, +40° 837.

TABLE II.—*continued.*

No.	Approximate Magnitude.	R.A. 1900·0.			Decl. 1900·0.		Annual Proper Motion. $\mu_\alpha \cos\delta$.	μ_δ.
		h	m	s	°	′	″	″
126	10·8	3	38	29	+41	8·1	−0·007	0·000
127	11·4	3	37	2	+41	7·6	+0·014	−0·020
128	11·8	3	41	55	+41	7·0	+0·040:	−0·030
129	10·2	3	37	26	+41	6·5	−0·004	−0·008
130	11·1	3	37	51	+41	6·3	−0·022	0·000
131	11·8	3	38	35	+41	6·1	+0·011	−0·005:
132	11·8	3	40	33	+41	5·8	−0·002	+0·018
133	11·4	3	37	28	+41	5·8	+0·008	−0·001
134	10·2	3	38	56	+41	4·7	+0·024	+0·008
135	10·2	3	35	39	+41	4·1	−0·053::	+0·034
136	11·1	3	41	57	+41	4·1	−0·003	+0·010
137	10·2	3	39	10	+41	3·9	−0·020	+0·004
138	11·4	3	37	18	+41	3·6	−0·018	−0·004
139	10·2	3	40	40	+41	3·4	+0·020	0·000
140	11·8	3	36	17	+41	3·3	+0·005	+0·008
141	11·8	3	40	50	+41	2·8	−0·003	−0·010
142	11·4	3	35	35	+41	2·3	+0·012::	−0·007
143	11·8	3	36	52	+41	2·3	−0·021	−0·005
144	11·8	3	36	43	+41	1·4	+0·002	+0·013
145	11·4	3	40	4	+41	1·1	−0·023	−0·022
146	11·4	3	41	23	+41	0·8	+0·010	−0·003
147	11·8	3	37	25	+41	0·9	−0·023	+0·032::
148	10·2	3	38	31	+41	0·5	−0·017	+0·010
149	11·1	3	37	19	+41	0·3	−0·019	−0·004
150	11·8	3	37	14	+41	0·1	−0·017:	−0·002
151	11·8	3	41	45	+40	59·0	−0·034	−0·005
152	10·2	3	38	29	+40	58·9	+0·017	+0·011
153	11·4	3	39	43	+40	58·9	−0·001	−0·011
154	11·4	3	39	26	+40	58·6	−0·010	+0·024
155	9·0	3	41	57	+40	58·3	+0·043	−0·002
156	11·8	3	38	9	+40	58·2	+0·056:	+0·007::
157	11·8	3	39	3	+40	57·5	−0·023::	−0·061::
158	10·5	3	38	28	+40	57·4	−0·009	+0·002
159	11·4	3	36	52	+40	56·4	−0·024	+0·009
160	11·8	3	39	6	+40	56·4	−0·023	+0·007

No. 136 B.D. +40° 840.
,, 155 ,, ,, 839.

TABLE II.—*continued.*

No.	Approximate Magnitude.	R.A. 1900·0.			Decl. 1900·0.		Annual Proper Motion. $\mu_\alpha \cos\delta$.	μ_δ.
		h	m	s	°	′	″	″
161	11·4	3	41	7	+40	56·0	+0·005	0·000
162	11·8	3	40	29	+40	56·0	0·000	−0·012
163	11·4	3	37	58	+40	55·4	+0·008	+0·001
164	11·4	3	36	20	+40	55·2	+0·020	−0·002
165	11·8	3	40	25	+40	55·0	+0·001	−0·012
166	11·4	3	40	46	+40	54·7	0·000	+0·031
167	11·1	3	35	53	+40	54·4	+0·006	−0·005
168	11·4	3	40	46	+40	53·7	−0·002	+0·015
169	10·8	3	36	34	+40	53·6	+0·010	−0·012
170	11·4	3	37	41	+40	53·6	−0·037	+0·005
171	11·8	3	36	28	+40	53·5	−0·018:	−0·016::
172	11·1	3	35	47	+40	53·0	+0·027	+0·001
173	11·4	3	37	31	+40	53·1	+0·033	+0·001
174	10·2	3	41	59	+40	52·8	+0·025	−0·004
175	11·8	3	38	29	+40	52·8	−0·014:	−0·008
176	11·4	3	40	12	+40	52·4	+0·038:	+0·016
177	11·1	3	39	10	+40	52·4	−0·022	−0·003
178	11·4	3	38	20	+40	52·2	−0·003	+0·009
179	10·2	3	40	17	+40	51·7	+0·001	−0·005
180	11·8	3	41	7	+40	50·9	+0·020:	+0·007
181	7·8	3	41	17	+40	50·9	−0·007	−0·017
182	11·8	3	40	40	+40	50·9	−0·002	+0·027
183	11·8	3	39	25	+40	50·9	−0·005	+0·030
184	11·1	3	36	41	+40	50·6	−0·003	+0·032
185	11·1	3	39	17	+40	50·7	+0·010	−0·018
186	11·4	3	39	55	+40	50·6	+0·004	−0·009
187	11·4	3	38	52	+40	50·3	−0·018	+0·008
188	11·8	3	39	34	+40	50·2	0·000	−0·017
189	10·2	3	35	40	+40	49·8	+0·043	+0·002
190	11·8	3	39	50	+40	49·5	+0·010:	+0·015::
191	11·4	3	39	50	+40	48·8	−0·001	+0·017
192	11·8	3	40	37	+40	47·8	+0·002	+0·030
193	11·8	3	38	39	+40	47·8	0·000:	−0·007::
194	11·4	3	41	57	+40	46·9	+0·011	−0·007
195	11·4	3	37	36	+40	47·1	+0·003	−0·002

No. 169 B.D. +40° 822.
,, 181 ,, ,, 834.

TABLE II.—*continued.*

No.	Approximate Magnitude.	R.A. 1900·0.			Decl. 1900·0.		Annual Proper Motion. $\mu_\alpha \cos\delta$.	μ_δ.
		h	m	s	°	′	″	″
196	11·8	3	37	3	+40	46·8	−0·022::	−0·036:
197	11·4	3	36	11	+40	46·8	−0·010	0·000
198	10·8	3	41	16	+40	46·7	+0·002	+0·004
199	10·8	3	40	32	+40	46·6	+0·005	−0·004
200	11·4	3	35	53	+40	46·4	+0·025	−0·020
201	11·8	3	41	10	+40	46·4	+0·001:	+0·015:
202	11·4	3	40	16	+40	46·5	+0·024	−0·015
203	11·4	3	41	10	+40	45·9	+0·011	−0·028
204	11·4	3	39	4	+40	45·4	−0·013	−0·013
205	10·5	3	40	56	+40	45·2	+0·020	−0·009
206	9·3	3	37	52	+40	44·9	−0·016	+0·013
207	11·8	3	39	42	+40	44·7	+0·020	−0·002
208	10·8	3	38	59	+40	44·5	−0·007	−0·006
209	9·3	3	41	13	+40	44·1	+0·004	+0·018
210	11·4	3	40	2	+40	44·0	+0·008	−0·001
211	11·1	3	39	17	+40	43·9	0·000	+0·019
212	11·1	3	41	4	+40	43·5	−0·002	−0·006
213	11·1	3	40	8	+40	43·2	−0·018	−0·009
214	10·2	3	39	2	+40	43·1	+0·014	+0·006
215	9·9	3	40	49	+40	42·6	−0·034	−0·017
216	11·4	3	36	40	+40	42·6	−0·025:	+0·017
217	11·4	3	36	46	+40	42·4	+0·019	−0·008
218	10·8	3	38	11	+40	42·2	−0·007	+0·001
219	9·9	3	35	38	+40	41·9	−0·004	+0·014
220	10·5	3	41	29	+40	41·9	+0·009	−0·021
221	11·4	3	36	36	+40	41·6	+0·017	−0·024
222	8·0	3	36	25	+40	40·5	+0·045:	−0·055:
223	11·4	3	36	41	+40	39·9	−0·010	0·000
224	10·5	3	36	2	+40	39·7	−0·027	+0·009
225	11·4	3	37	44	+40	39·2	−0·005	−0·011

No. 206 B.D. +40° 827.
,, 209 ,, ,, 833.
,, 222 ,, ,, 821.
,, 223 ,, ,, 823.

TABLE II.—*continued.*

No.	Approximate Magnitude.	R.A. 1900·0.			Decl. 1900·0.		Annual Proper Motion. $\mu_\alpha \cos\delta$.	μ_δ.
		h	m	s	°	′	″	″
226	11·4	3	40	37	+40	38·9	+0·001	−0·008
227	11·4	3	37	40	+40	38·6	−0·028.	+0·002
228	9·0	3	41	47	+40	38·3	+0·023	−0·048
229	10·5	3	37	37	+40	38·1	−0·011	+0·008
230	10·2	3	39	23	+40	38·1	−0·008	−0·020
231	9·3	3	41	52	+40	37·4	+0·019	−0·022
232	8·5	3	38	6	+40	37·4	−0·013	+0·009
233	10·5	3	40	1	+40	37·3	+0·013	−0·002
234	11·8	3	40	23	+40	37·3	+0·005::	+0·001
235	11·4	3	38	17	+40	37·2	+0·027:	−0·013
236	9·6	3	36	23	+40	36·8	0·000	−0·006
237	11·8	3	39	50	+40	36·5	+0·017::	−0·001::
238	11·8	3	40	20	+40	36·4	+0·014::	+0·025:
239	10·5	3	38	14	+40	36·4	+0·004	−0·006
240	11·8	3	39	5	+40	36·3	+0·020	0·000
241	8·7	3	36	51	+40	35·8	+0·048	+0·015
242	11·4	3	38	39	+40	35·7	−0·002	+0·003
243	10·8	3	39	38	+40	35·1	+0·014	−0·019
244	11·8	3	35	40	+40	34·8	−0·024	+0·058
245	11·4	3	37	22	+40	35·0	−0·018	+0·029
246	11·1	3	38	42	+40	34·9	−0·001	−0·006
247	11·8	3	39	11	+40	34·7	−0·031	+0·007
248	11·4	3	35	58	+40	33·5	−0·004	+0·037
249	11·8	3	37	50	+40	33·3	−0·010:	+0·004:
250	11·4	3	40	23	+40	32·1	+0·010	0·000

No. 228 B.D. +40° 836.
,, 231 ,, ,, 838.
,, 232 ,, ,, 828.
,, 236 ,, ,, 820.
,, 241 ,, ,, 825.

Radcliffe Observatory, Oxford:
1913 *May.*

ADDENDUM.

At the time of writing the above, the material available for deducing the proper motion of any of these stars from existing catalogues was, except for Σ443, so scanty that it did not appear worth while to include in the paper a comparison of such proper motions with the corresponding quantities deduced from this plate

(37B). Since then I have been in correspondence with Professor A. S. Donner, Director of the University Observatory, Helsingfors, who has kindly placed at my disposal the places of nine B.D. stars from the *Astrographic Catalogue*. The star-places are contained in a volume of the Helsingfors Astrographic Zone, which is at present passing through the press. They correspond to the epoch 1896·0.

The accompanying table exhibits in parallel columns the values of $\mu_\alpha \cos\delta$ and μ_δ deduced from the catalogues and from the single Oxford plate. In the last column are given references to the catalogues compared.

Comparison of Proper Motions deduced from Catalogues with Corresponding Quantities deduced from the Oxford Plate 37B.

No. in Table II.	$\mu_\alpha \cos\delta$. Catal.	$\mu_\alpha \cos\delta$. Radcliffe Photo.	μ_δ. Catal.	μ_δ. Radcliffe Photo.	Relative Weights of Cat. P.Ms.	Catalogues compared.
12	+0″·01	−0″·01::	−0″·02	0″·00::	10	Lal.*, Grb., W.B.*, Rad. 45, A.G., Gr. 90, Hels.
37	− ·06	− ·07	+ ·05	+ ·07	6	W.B.*, Brus. 65, A.G., Hels.
53	− ·01	·00	− ·02	− ·02	6	W.B., Bonn VI.*, A.G., Hels.
75	− ·02	·00	·00	·00	4	W.B., A.G., Hels.
88	+ ·01	+ ·05	·00	+ ·02	4	Lal.*, W.B., A.G., Hels.
206	+ ·02	− ·02	+ ·04	+ ·01	2	Bonn VI.*, Bonn Veröff. 9*, Hels.
222	+ ·01	+ ·04:	− ·03	− ·05:	4½	Lal.*, Bonn VI., A.G., Paris, Hels.
232	− ·02	− ·01	+ ·02	+ ·01	4	Lal.*, W.B.*, Paris, A.G., Hels.
241	+ ·03	+ ·05	·00	+ ·01	2½	Lal.*, A.G., Paris, Hels.

Radcliffe Observatory, Oxford:
1913 *June* 24.

The Spectra of Novæ and the Pressure Effect.

Professor W. G. Duffield, D.Sc.

The spectra of temporary stars have been discussed on former occasions † with special reference to the part played by the pressure of the atmosphere surrounding a Nova, but as the investigation of sources of light subjected to high pressure has been considerably extended in recent years, a revision of the subject may be now, I think, undertaken with profit, especially as observations of the spectra of Novæ are rapidly accumulating.

* Single observations.

† Wilsing, *Astrophysical Journal*, x. 113, 1899; Lockyer, *Astrophysical Journal*, xv. 190, 1902; Hale and Kent, *Publications of the Yerkes Observatory*, vol. iii. pt. ii., 1907.

In the following examination will be found the point of view of the worker in the physical laboratory, the measurements of others upon the spectra of Novæ being necessarily taken without criticism. The treatment of the subject is designed to be of value in the future even if the data upon which I have based my present arguments should prove to be erroneous; this is why I have dwelt at some length upon the qualitative agreement between a certain aspect of the spectrum of a Nova and that of a source under pressure, though I subsequently show that existing quantitative evidence is against it. For the same reason I have been anxious throughout to bring forward more or less categorically the arguments for and against the various phenomena common to the two spectra being causally related.

At the end of the paper I have set down those conditions which, in my opinion, must be fulfilled by a system giving a spectrum of the type under consideration.

The subject is dealt with under the following headings:—

1. The doubling of the lines.
2. The broadening of the lines.
3. The intensity curves and structure of the broadened lines.
4. The reversal and relative intensities of the lines.

1. *The Doubling of the Lines.*

The phenomenon thus referred to is the occurrence upon the more refrangible edge of many broad bright lines of a broad absorption line.

It is necessary to inquire whether the bright and dark components are produced by (*a*) the same influence, or (*b*) different influences.

Qualitative Discussion.—(*a*) If both are due to the same cause, pressure naturally suggests itself, since the phenomenon can be qualitatively reproduced, as Wilsing* showed, by a spark discharge between metal poles under water. Wilsing ascribed this phenomenon to the effect of pressure on indirect evidence that a spark produced under water is under high pressure; since then lines presenting this appearance have been shown to be readily produced in certain spark † and arc ‡ discharges in compressed air.

As the elements, chiefly hydrogen and helium, which possess this characteristic feature in the stellar spectrum, are not easily examined under pressure in the laboratory, it is necessary to consider whether there is any evidence indicating that the hydrogen and helium lines are of the same general nature as those lines of other elements which are known to show doubling under pressure.

In the iron arc lines which behave thus broaden easily and suffer large displacements towards the red with increase of pres-

* Wilsing, *loc. cit.*
† Hale and Kent, *loc. cit.*
‡ Duffield, *Phil. Trans. Roy. Soc.*, ccviii. 111, 1908.

sure; the reversal is not necessarily apparent at atmospheric pressure, the absorption line developing as the pressure is increased, but at higher pressures usually becoming less marked. As in the spectra of Novæ the reversed part varies considerably in width on different occasions, being sometimes narrow in comparison with the bright line upon which it is superposed and sometimes nearly half its width. Such lines all belong to what I have called Group III., these being displaced twice as much as the lines of Group II. and four times as much as those of Group I. It has not been determined to what recognised spectrum series (if any) such lines belong, but it is to be noted that in the case of elements possessing lines belonging to a recognised series such lines are characterised by larger displacements than the non-series lines.* There is thus some reason for regarding the lines in the iron spectrum which behave in this way as corresponding to one of the recognised series. This cannot be pressed far, but it suggests that it is not unreasonable to look for a similar behaviour in the series lines of hydrogen and helium which occur in the spectra of temporary stars.

It is important to note that A. S. King,† who employed an electric furnace for the production of spectra under pressure, did not observe any lines to be unsymmetrically reversed when the furnace was evenly heated, although the strongest of all such lines that have been observed in the arc occur in the region of the spectrum which he examined. This difference between arc and furnace spectra is due, I think, to the fact that in the former there is a gradient of pressure, density, or temperature in the luminous vapour, whereas in King's experiment the vapour should be nearly homogeneous. A later experiment of King's ‡ tends to support this view, because he finds that by heating the middle of the carbon tube to a temperature higher than that of the ends a symmetrical reversal may be converted into one that is unsymmetrical. If the pressure effect proves to be capable of explaining the doubling of the lines in a typical Nova spectrum, we have evidence that the source is not homogeneous like the vapour in King's original experiments, but that its density gradient and temperature gradient more closely resemble those of the arc; conditions which might also be realised in an advancing shell or sphere of luminous vapour in which the density of the foremost portion is less than that of the main mass.

Quantitative Discussion.—We have hitherto been concerned only with relative values of the displacement of bright and dark lines, and the phenomenon has been found to resemble closely that obtained by subjecting certain sources of light to pressure, but it was long ago pointed out that if the absolute values of the displacement in a Nova's spectrum be taken into account, the bright component will be usually found to be displaced towards the red,

* Duffield, *Phil. Trans. Roy. Soc.*, ccix. 205, 1908.
† King, *Astrophysical Journal*, xxxv. 183, 1912.
‡ King, *ibid.*, xxxvii. 119, 1913.

whereas the dark component is invariably displaced towards the more refrangible end of the spectrum relatively to its normal position. This is contrary to the usual direction of displacement due to increased pressure, and far greater in amount than can be accounted for by any possible reduction of pressure, if, as it appears, a linear relation holds down to the lowest pressures.* This is a serious difficulty in the pressure interpretation of the spectrum, and I do not think that the fact that an increase in pressure displaces some lines towards the violet assists matters much; in the rare instances in which such lines are reversed, the absorption line, as would be expected, is less displaced to the violet than the bright part, thus providing the converse of this particular feature of the Nova.

Combination of Radial Velocity and Pressure.—On the basis of our existing knowledge of displacements towards the violet under the influence of pressure, it is impossible to refer the phenomena to pressure alone. It is, however, important to inquire whether the combination of a large radial velocity with a high pressure might not reproduce this particular feature of the observed spectrum.

I have shown elsewhere † that when unsymmetrically reversed lines are produced by pressure the reversed part is displaced approximately half as much towards the red as is the bright line when unaffected by reversal; that is to say, it is not the most intense part of the bright line which is subjected to absorption. The intensity curve of a line affected by pressure has the form represented by the full line :—

The dotted line represents the intensity curve of the broadened and displaced line when there is no absorbing vapour.

N′ is the normal position of the line.

D is the position of the dark reversal.

T is the true maximum of the bright line if unaffected by reversal.

O is the observed maximum of the bright line.

This diagram may be regarded as representing the effect of radial velocity upon a line also affected by pressure, if it be supposed that N is the position of the comparison line at atmospheric pressure, and that N′ is the position of the line displaced to the violet by motion, but unaffected by pressure. Qualitatively the diagram illustrates the intensity curve of the typical line in a Nova's spectrum.

* Mohler, *Astrophysical Journal*, iv. 175, 1896.

† Duffield, *Phil. Trans. Roy. Soc.*, ccviii. 111, 1908.

In this diagram $NN' =$ Displacement due to motion;
$ND =$ Measured displacement of dark line;
$NO =$ Measured displacement of bright line;
$N'D =$ Displacement of dark line due to pressure;
and it is evident that $N^1D = NN^1 - ND$ . . . (1).

The obvious difficulty of reconstructing the dotted intensity curve from the observations of ND and NO points to the reversed parts as providing the best means for dealing with the question.

Pressure Displacement and Wave-Length.—Before proceeding further it is necessary to draw attention to another feature of the spectrum of a body subjected to pressure. It is well known that motion in the line of sight displaces each line by an amount which is proportional to its wave-length, and it was at first thought that the pressure displacement varied in precisely the same way and that the Doppler and pressure effects were indistinguishable in this respect; but more recent work has shown that the pressure displacement varies with a higher power of the wave-length than the first, being at least as high as the second power and probably as high as the third, but there is difficulty in distinguishing its exact rate. This result was first established for copper * and gold † by the writer, and has been confirmed by Gale and Adams ‡ and King § for iron, and more recently by the writer for nickel. This, if confirmed for other metallic vapours and gases, provides a criterion for distinguishing between the two effects if the displacements of the lines of a single element can be obtained with sufficient accuracy. If the displacement varies as the first power of the wave-length, radial motion is suspected; if the displacement varies with a higher power, the pressure effect is inferred.

We are now in a position to deal with equation (1) if we take from the following section page 636 the fact derived from measurements of spectrograms of Novæ, namely that the displacements of the dark lines are proportional to their wave-lengths:—Thus $ND = b\lambda$. We also know that NN' which is due to motion can be written $NN' = a\lambda$, and that $N'D$ is proportional to $c\lambda^n$ where n is not very different from 2 or 3, a, b, and c being constants.

Equation (1) becomes $c\lambda^n = a\lambda - b\lambda$ which is dimensionally impossible.

Hence it is impossible for the measured displacement of the dark lines to be reproduced by the combined effect of pressure and radial motion, whatever may be the origin of the bright bands. The answer to question (a) is therefore in the negative as far as pressure is concerned, even in the most favourable case when radial motion is imagined to displace the line as a whole, and in spite of the suggestive qualitative resemblances to a pressure effect.

Since a rational solution of the equation is given by $c = 0$, we

* Duffield, *ibid.*, ccix. 205, 1908.
† Duffield, *ibid.*, ccxi. 33, 1910.
‡ Gale and Adams, *Astrophysical Journal*, xxxv. 10, 1912.
§ King, *ibid.*, xxxv. p. 183, 1912.

see that $a = b$, and that the displacement of the dark line can be explained (as was at once apparent) by radial velocity alone. As we shall see under the next heading that the bright band is not displaced in accordance with a radial velocity (though this factor may be concerned in the broadening of the line), we conclude that the bright and dark displacements are not produced by the same cause, whether pressure or motion in the line of sight.

The Dark Bands.—If the bright and dark components are of separate origin, we must treat of them separately. The dark lines can be divided into two groups—those that are very broad like those occurring on the violet side of the hydrogen lines, and those that are fine like the dark H and K lines on the bright calcium band and the dark D lines on the bright sodium band.

These will be subsequently referred to as dark bands and dark lines respectively. The former are frequently double in the case of the hydrogen and helium lines. Their displacements in the spectrum of Nova Geminorum * are given in the following table, in which d_1 and d_2 are the measured displacements of the two dark bands. Doubtful identifications are included in brackets. Throughout this paper all displacements are in Ångström units.

TABLE I.

Dark Hydrogen Bands.

	λ.	d_1.	$\frac{d_1}{\lambda}$.	d_2.	$\frac{d_2}{\lambda}$.
H_ϵ	3970·25	−11·2	−28	−19·7	−50
H_δ	4101·85	−11·3	−28	−19·3	−47
H_γ	4340·66	−12·3	−28	−20·7	−48
H_β	4861·49	−13·8	−28	−23·7	−49
H_α	6563·04	−17·8	−27	...	...

Dark Helium Bands.

4143·92	(−10·5)	(−25)	(−20·3)	(−49)
4437·72	(−8·2)	(−19)	(−19·1)	(−43)
4471·65	−11·0	−25	(−19·4)	(−43)
4922·10	−11·0	−22	−21·5	−44
5015·73	−11·5	−23	−21·0	−42

In the third and fifth columns are given the values of the displacements divided by the wave-length. These are sufficiently in agreement to indicate that the displacement is proportional to the wave-length for each dark band, and it is further clear that within close limits the hydrogen and helium bands possess the same rate of displacement.

Nova Persei showed a similar relationship, as may be seen by Table II., in which Campbell and Wright's measurements † are recorded.

* Adams and Kohlschutter, *Astrophysical Journal*, xxxvi. 293, 1912.

† Campbell and Wright, *ibid.*, xiv. 269, 1901.

Table II.

Element.	Wave-length.	d.	$\frac{d}{\lambda}$.
Calcium K	3933·83	−17	43
,, H	3968·63	−16	40
Hydrogen H_δ	4102·00	−19	46
,, H_γ	4340·53	−19	44
,, H_β	4861·57	−24	49
Sodium D	5896·3	−27	46

The same observers chronicle another more accurate and complete set of displacements of the hydrogen lines in Nova Persei.

Table III.

Line.	Wave-length.	d.	$\frac{d}{\lambda}$.
H_ζ	3889·15	19·64	50
H_ϵ	3970·33	19·98	51
H_δ	4102·00	20·79	50
H_γ	4340·53	22·91	51
H_β	4861·57	24·5	53

Comparing the corresponding hydrogen and helium displacements in Nova Geminorum, we see evidence of a common ratio of displacement of the lines of these elements, and the same fact emerges in the spectra of hydrogen, calcium, and sodium in the spectrum of Nova Persei.

It has been shown that this cannot be due to a combination of pressure and velocity, but that it must be due to velocity alone.

Hence it appears that in these two Novæ the different elements have at any time practically the same velocities as far as their dark bands are concerned, and that when the dark bands are doubled the velocities of corresponding dark bands also agree, the two values being such that one is approximately double the other. Unfortunately the displacements of bands due to other elements are rather doubtful, but the displacements of the dark bands referred to iron and titanium* suggest that they share in the common velocity.

The measurements of the fine dark lines on the bright bands of calcium and sodium are not in good agreement with one another, but in Nova Persei they indicate velocities of the vapour to which they are due varying from +30 to +36 kms. per second (which reduce after correction to velocities varying from +0·5 km. to +6 kms. per second), and in Nova Geminorum varying from +28·5 to +47·4 kms. per second (which reduce to velocities varying from +1·0 km. to +18·0 kms. per second, the mean being about +10 kms.).

Campbell and Wright and Adams and Kohlschutter take the velocities indicated by these fine dark lines as the radial motion

* Curtis, *Monthly Notices*, lxxii. 511, 1912.

of the star as a whole, and have corrected their tables of wave-lengths for these velocities. It appears to me to be wise to keep distinct for the present the different sets of velocities indicated by the spectrum, bearing in mind that those calculated for the broadened bands are the most important as they refer to the bulk of the absorbing vapour.

An interesting plate was obtained by Adams and Kohlschutter on March 30, on which each hydrogen line possessed four dark components. The displacements of these from the normal position are given in the following table, together with the ratios of the values of $\frac{I}{d^2}$ referred to the most displaced component.

TABLE IV.

	A.	B.	C.	D.
	Values of Displacements = d.			
$H\delta$	(11·4)	14·3	21	24·6
$H\gamma$	12·2	14·4	20·9	24·8
$H\beta$	(13·8)	15·0	23·6	28·3
	Values of $\rho = \frac{I}{d^2}$.			
$H\delta$	(4·7)	3·0	1·4	1
$H\gamma$	4·1	3·0	1·4	1
$H\beta$	(4·1)	3·5	1·4	1

The values in brackets are taken from a different plate, as the component marked A was not measured on the plate under discussion. Reference will be made later to the values of $\frac{I}{d^2}$ obtained above

The Bright Bands.—The displacements of the bright components of the hydrogen bands are given in the following table, which relates to Nova Geminorum.* In the fourth and fifth columns are given the values of displacements divided by the first and second powers of the wave-length. It is apparent that the agreement is greater for the constant depending upon a relation between the displacement and the square of the wave-length, a fact to which Adams and Kohlschutter draw attention.

TABLE V.

	λ.	d.	$\frac{d}{\lambda}$.	$\frac{d}{\lambda^2}$.
H_ϵ	3970·25	+1·25	+31	+79
$H\delta$	4101·85	+1·35	+33	+80
$H\gamma$	4340·66	+1·74	+40	+92
$H\beta$	4861·49	+1·81	+37	+77
$H\alpha$	6563·04	+3·79	+57	+87

* Adams and Kohlschutter, *loc. cit.*

Unfortunately this is almost the only concordant evidence that I have been able to find relating to the rate of displacement of the bright bands. In Nova Persei the measurements are very vague and irregular, and are not considered very accurate by the observers,* the limits of error being several Ångström units. The following are the displacements of the bright hydrogen bands in Nova Persei for three different dates in the year 1901 :—

TABLE VI.

	λ	d Feb. 25.	d March 13.	d March 18.
H_ζ	3889·15	...	+7	+3
H_ϵ	3970·25	...	+4	+5
H_δ	4101·85	−8	+3	+6
H_γ	4340·66	−6	+1	+6
H_β	4861·49	−9	+1	+7

It is evident that we must rely upon the Nova Geminorum measurements which favour a displacement varying with the second power of the wave-length. It should also be noted that this rests, to a considerable extent, upon the measurement of the H_a line which the observers state was not often obtained satisfactorily upon their photographs. These results support a pressure rather than a Doppler effect, because the stellar and pressure spectra agree in requiring a displacement which varies more rapidly than the first power of the wave-length. It should be remarked that this evidence is not conclusive, because the pressure displacement seems to vary with the cube and that of the bands with the square of the wave-length ; but it is not so easy to distinguish between these two rates of displacement as between variations with the first and second powers of the wave-length. Consequently, using the information at our disposal, we must regard the pressure influence as favoured.

The helium bright bands have also been measured in the spectrum of Nova Geminorum, but their displacements show little obvious regularity. Adams and Kohlschutter state that on the whole they appear to be displaced proportionally to the wave-length, but examination of their data shows that the irregularities of the displacements are such that there is little to choose between variations with the first and second powers of the wave-length. If, however, a table be constructed showing the values of $\frac{d}{\lambda}$ and $\frac{d}{\lambda^2}$ for lines of the same spectrum series, rather more regularity appears. In Table VII. these values are given under the heading indicating the series to which the lines belong :—

* Campbell and Wright, *loc. cit.*

TABLE VII.

Values of $\frac{d}{\lambda^2}$.

Principal Series Parhelium.	1st Sub-series Parhelium.	1st Sub-series Helium.	2nd Sub-series Parhelium.
+150	+144	(+228)	+77
	(−36)	+34	+62
	+157	+34	
	(−30)		

Values of $\frac{d}{\lambda}$.

+75	+60	(+95)	+35
	(−16)	+15	+36
	+77	+17	
	(−20)		

It is possible that the greater regularity thus obtained is apparent rather than real, and I realise that stress should not be laid upon it on account of the doubt expressed by the observers about the accuracy of the readings; but it is worth pointing out with caution (that it may be considered in future) that (omitting the negative values, since these are discordant with both a pressure and a Doppler explanation, and one other value which is included in brackets) the displacements are in the ratio of 34 : 70 : 150 for $\frac{d}{\lambda^2}$ which is in sufficient agreement with the 1 : 2 : 4 relationship found for different groups of lines of other elements when subjected to pressure to give a bias to that influence as their origin, though the spectrum series giving those ratios are not (according to Humphreys *) in the order here indicated.

It must be admitted that the information afforded by the displacements of the bright bands is not so reliable as that provided by the displacements of the dark bands, since it is always possible that the former are affected by absorption, and it is a matter of great importance whether their centres or brighest parts are measured. In the event of subsequent observations indicating a linear displacement (which I do not anticipate), it would involve very different velocities for the vapour responsible for the dark and bright bands respectively.

2. *The Broadening of the Spectrum Lines.*

Rossi † has shown that the hydrogen lines maintain their relative widths independently of the pressure, but that the absolute value of the widths is proportional to the pressure. The measurement of the widths of the hydrogen lines in a Nova might be expected to give a measure of the pressure to which this gas is subjected, but the electrical conditions of the source have so large

* Humphreys, *Astrophysical Journal*, vi. 428, 1897.
† Rossi, *ibid.*, xxxiv. 299, 1911.

an influence upon the width of a line that this can give no reliable information upon the pressure about a Nova. Qualitatively, the broadening of the lines is not unfavourable to the pressure explanation, as it is a feature of all pressure experiments, and as it is true in general, I think, that while a dense vapour may be made to give a comparatively narrow line by appropriate excitation, a rare vapour cannot be made to emit a broadened line.

It is curious that measurements of the widths of the bright hydrogen lines in the spectrum of Nova Geminorum * give a ratio which is the reverse of that found by Rossi, the H_α line being the broadest in the Nova spectrum, and the narrowest in Rossi's terrestrial determination as the following measurements show :—

	$H\delta$.	$H\gamma$.	$H\beta$.	$H\alpha$.
Width per atmosphere . .	49·3	40·4	24·7	8·7
Width in Nova . . .	22·3	23·8	28·2	38·8

In the first case the broadening is proportional to the inverse third or fourth power, and in the latter to the first power of the wave-length. This suggests that the broadening is due to a Doppler effect, since vapour whose velocity varies over a range δv will give spectral lines whose widths are proportional to $\lambda \cdot \delta v$. If this holds good, it demonstrates that the vapour in motion is not very dense, because if it were, the more refrangible lines would be so increased in width that the linear relationship would be masked.

3. *The Intensity Curves and Structure of the Broadened Lines.*

In the spectra of Novæ it has been remarked that the hydrogen bands are frequently of a complicated nature, consisting of alternations of bright and dark lines of irregular widths. A pronounced structure which resembles this has been observed upon the wings of certain lines produced in a silver arc when subjected to pressures of a few atmospheres.† The wings of the silver lines under pressure consist entirely of fine lines spaced very irregularly : these lines extend over twenty Ångström units under a pressure of five atmospheres, and over about five times that range under twenty atmospheres. As the pressure is further increased they ultimately fill the space between the series lines and give rise to a banded spectrum. It is not known whether hydrogen, if submitted to sufficient pressure, would show this structure, but it may be remembered that the silver lines presenting this phenomenon belong to the first subordinate series, which is the spectrum series to which the hydrogen lines are referred. As far as I am aware, pressure is the only influence capable of accounting for this phenomenon.

It is also of special interest in this discussion to note that the maximum of intensity of a bright line in the spectrum of a Nova

* Adams and Kohlschutter, *loc. cit.*

† Duffield, *Phil. Trans. Roy. Soc.*, 211, p. 33, 1911. See Plate 1.

is nearer its more refrangible edge * than the red edge, and that this is a marked feature of the distribution of intensity in a line affected by pressure, as may be seen by reference to the dotted line in the diagram on p. 634. If the broad band were wholly due to line-of-sight motion, the brightest part would mark the maximum ordinate of the velocity-distribution curve; the maxima of different lines should then be displaced proportionally to the wave-length, which is found not to be the case.

4. *The Reversal and Relative Intensities of Lines.*

In the discussion of the pressure effect by Hale and Kent to which reference has already been made, reasons are given for rejecting Wilsing's theory. My own views upon the displacements of the lines have already been expressed in detail, but a word is due regarding two other objections which they make. Their contention is that under the high pressure, which in their opinion must exist in Novæ if it is to assist in explaining the spectrum, the spectrum should be one of absorption lines upon a continuous background; this is based upon experiments with spark discharges in air under pressure and under water, and is not upheld by experiments upon arc discharges in air under still higher pressures, the reversals often becoming less numerous at one hundred atmospheres than they are at ten or twenty atmospheres. The reversal phenomena observed by Hale and Kent are thus not an *essential* feature of a spectrum under pressure, but depend upon the nature of the discharge.

The same may be said of two other points which should be discussed in this connection; the first is the possibility that all spectra may become continuous if subjected to sufficient pressure, like the spectrum of the silver arc under two hundred atmospheres; the second is the change in the relative intensities of the lines which occurs with increasing pressure. Both these phenomena depend too much upon the nature of the electrical discharge to form criteria for testing the spectra of Novæ—the continuous spectrum, for example, changes back to the line spectrum at the instant when the arc is extinguished, and the relative intensities of the lines in King's furnace tube are different from those obtained in the arc discharge, though both may be under the same pressure. Thus none of the effects discussed under the present heading are capable in themselves of settling the question under consideration.

Summary.

It has been shown that the phenomenon of doubling cannot be explained quantitatively by the effect of a dense atmosphere about the Nova, even when the star as a whole is regarded as being in motion in the line of sight, and in spite of certain qualitative agreements.

* Vogel, *Astrophysical Journal*, xiii. 17, 1901; Curtis, *Monthly Notices*, lxxii. 511, 1912.

When the bright components were treated separately it was found that on the whole their displacements were consistent with a pressure rather than a Doppler effect, and the further evidence that their structure and intensity curves are capable of imitation in arcs under pressure supports this conclusion.

But it has also been shown, upon the basis of Rossi's experiments, that the widths of the lines cannot be explained by pressure, and that they are conformable with line-of-sight motion over a wide range of velocities. If the same vapour is responsible for both displacement and broadening, the conditions must be such that a Doppler broadening can take place about a pressure-displaced position, which might arise from vapour under pressure being in motion in all directions. Alternatively, the displacement and broadening may be due to different vapours, one mass being subjected to pressure and the other being in motion, the appearance of the bright band being due to a superposition of the two effects. But if motion be the cause of the broadening of the lines, the evidence afforded by their structure and intensity curves must be discounted.

On the other hand, the dark components are displaced to the violet by amounts proportional to their wave-length, which indicates that they are due to line-of-sight motion; if they are double, each component obeys this law. The evidence points to the velocities of the absorbing vapours being approximately the same whatever their nature. Moreover, this common velocity is not very different in different temporary stars. Whenever in a Nova the same kind of vapour appears to possess two velocities, one of them is approximately double the other. These velocities are always large, being of the order of one five-hundredth part of the velocity of light. From the existence of a few fine dark lines with much smaller displacements towards the red, there must exist a small stratum of the vapours of calcium and sodium with smaller velocities in the opposite direction.

The conditions which must be satisfied by a Nova are thus such that the bright lines are due partly to vapour subjected to pressure, and partly to vapour (possibly the same vapour) in motion with variable velocities, and the dark lines due to vapour which is in a state of motion toward the Earth.

Of the systems which have been suggested at various times to account for the spectrum of a Nova, the following conform in certain respects with these conditions:—(*a*) One in which a dense star giving a bright line spectrum is behind a second cooler gaseous star advancing towards the Earth. The improbability of such a phenomenon repeating itself in all temporary stars will be readily conceded, and the complexity of the system of stars necessary to account for the two velocities observed in the absorbing vapour of the Nova would drive an explanation upon these lines to an absurdity.

(*b*) A system resembling that of the Sun is suggested by Deslandres,* who examined the integrated light of the Sun and found that it could be regarded as giving a bright line spectrum of

* Deslandres, *Comptes Rendus*, cliv. 1321, 1912.

calcium, the H and K lines being slightly displaced to the red and being accompanied by dark lines on their more refrangible edges. Deslandres considers the dark lines to be due to absorption produced by uprushes of vapour in the prominences, and the displaced bright lines to the descent of vapour; and suggests that this is an illustration upon a small scale of what is occurring in stars of the Nova type. This system, however, would necessitate a linear rate of displacement of the bright bands with wave-length which is contrary to the evidence. The theory is further open to the objection that the broad bright bands are by no means typical of luminous vapour in a medium where the rarity is such as must exist at so great a distance from the star that the enormous initial velocity of the vapour has been reduced to zero, unless we further suppose that the range of its velocities before and after coming to rest accounts for the observed broadening; moreover, it requires that the intensely hot rising vapour should be absorbing light, and that the descending vapour, which is cooler, should be emitting it; the converse would be expected, but as it is the temperature gradient which determines whether absorption occurs or not * a definite pronouncement is difficult. I incline to the opinion that a subsequent suggestion of Deslandres, that the spaces between the fissures which emit the prominences are under high pressure, and may be partly responsible for the displacement of the bright lines to the red, is more conformable with the spectral evidence. A dense bright-line star would supply the bright bands displaced to the red, and the top of the prominences might, as suggested above, supply the broadening, and the base of the prominences might supply the dark absorption bands displaced to the violet if the velocity of the vapour be supposed sufficiently great, though the observed regularity in the displacements of the dark bands is greater than would, I think, be expected if prominences, whose velocities are known to be extremely variable, are their cause. The doubling of the dark bands constitutes a difficulty here as it does in most theories.

(*c*) It is also questionable whether such large velocities could exist without the vapour being projected beyond the gravitational influence of the star (this depends upon the total mass of the star), in which case it has been suggested that it would travel outwards as a shell or sphere of advancing gas. It is true that such shells might provide the absorption that is observed, but the difficulty here lies in explaining why the velocities of all elements of which we have knowledge are the same. On the simplest assumption these velocities should be determined by the rates of diffusion of the gases. They should thus be proportional to $\left(\frac{T}{\rho}\right)^{\frac{1}{2}}$, where T is the absolute temperature and ρ the molecular density. The necessary condition for equality of velocities is that $T \propto \rho$, which might be effected if the high

* Schuster, *Astrophysical Journal*, xxi. p. 1, 1905.

temperature is produced by the conversion of a common translational velocity into molecular agitation, a class of phenomenon of which that suggested by Bickerton might be a special case. Alternatively the elements may have a common temperature and be thoroughly mixed before being subjected to projection, but it is unlikely that this common velocity would be maintained beyond a certain point at which diffusion velocities would be superposed upon it. The two or more values of the dark absorption bands again constitute a difficulty; it might be expected that the foremost part of the advancing shell would be sufficiently bright to give a displaced line in the centre of the reversed part, but Adams and Kohlschutter give reasons for believing that a new component is formed on the violet side of the previously existing dark band, which indicates another absorbing mass with a different velocity. On the diffusion theory this must be due to either a different temperature or different atomic aggregation. If the latter be conceived as possible, the molecular density is given by $\frac{1}{d^2}$. This is shown to possess values bearing the approximate ratios 1 : 1·4 : 3 : 4 on a certain photograph. Whether these are fortuitous future observation must decide, but there is an unexpected approximation to an integral relationship which could not easily be explained by eruptive prominences. The fine dark lines appear to be associated with the central body and to represent quiescent absorbing vapour, their displacements representing the velocity of that body as has already been suggested.

Hence, of the proposed systems we see that a bright-line star of moderate density might account for bright bands displaced to the red, and that the absorption bands might be due to gigantic uprushes of vapour with enormous velocities either resembling those in the Sun in which the projected material returns to the photosphere, or possibly giving rise to expanding spherical shells of absorbing vapour. Both might conceivably exist simultaneously; the latter seems in accordance with the photographs of the subsequent nebulosity about a Nova, the motion of the projected vapour in the former seems capable of explaining the broadening of the bright bands. Such a system seems able to supply the essential features of a Nova's spectrum as far as pressure is concerned. It labours under the difficulty of accounting for the huge velocity of the vapour, which, if due to diffusion, would indicate a temperature of almost inconceivable magnitude, but it may be that the vapours are ionised, in which case electrical forces may partly account for the high velocities.

In conclusion, it may be remarked that the displacements of the nebular lines in a Nova's spectrum, which apparently carry on the traditions of the hydrogen vapour, are not in accordance with the propagation of a luminosity through a pre-existing nebula.

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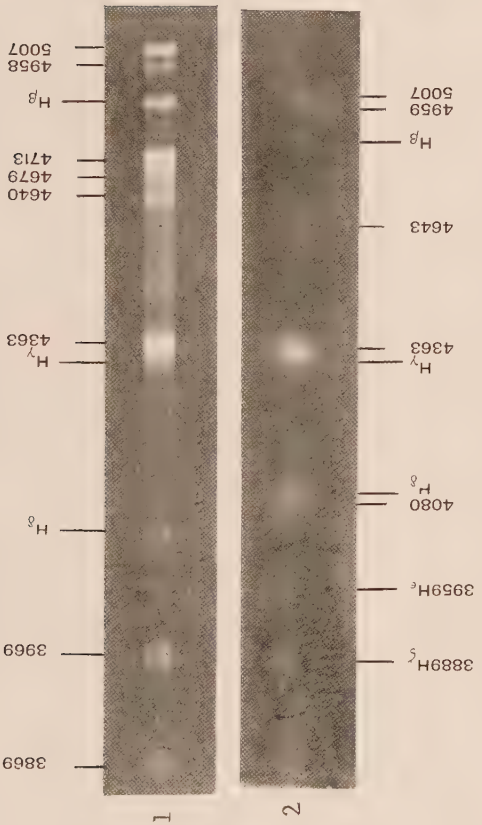
Reading.

The Spectrum of Nova Geminorum 2, 1912 *April and* 1913 *February–April.* By the Rev. A. L. Cortie, S.J. (Plate 21.)

Seven plates of the spectrum of Nova Geminorum 2 were secured with the Hilger single compound prism attached to the 15-inch Perry memorial equatorial in 1912 on the dates March 28, April 1, 4, 10, 11, 23, 24. At the beginning of the present year the 6-inch Dallmeyer portrait lens, presented some years since to the observatory by Mr. E. T. Whitelow, F.R.A.S., was furnished, through the generosity of the same donor, with a 6-inch objective prism of 19° angle which was made especially for the purpose by Mr. T. Thorp, F.R.A.S. The camera is attached to the tube of the 15-inch equatorial, which is used as a guiding telescope when faint objects are being photographed. The prism does full justice to the skill of its maker, and experimental exposures made on brighter stars gave a spectrum in excellent definition on every part of a quarter plate from H_α to H_η. A spectrum of the hydrogen series of α Lyræ has been utilised for the construction of a wave-length curve for the instrument. Six photographs of the spectrum of the Nova have been obtained on 1913 February 27, March 7, 11, 24, and April 4, 5. The best of these photographs is that of March 11, taken with an exposure of two hours on an Edwards isochromatic plate.

From Dr. Rambaut's measures the mean magnitude of the Nova for the series of plates in 1912 was 6·5, varying from 6·2 on March 28 to 6·8 on April 24. Mr. C. F. Butterworth has kindly furnished me with estimated magnitudes for the period nearly covering the second set of observations. The mean magnitude of the star was 8·8 and only varied 0·4 magnitude. All the measures on the plates were made on a Hilger measuring machine by myself. Of the 1912 spectra, that selected for measurement was the plate of April 24, of which a preliminary note has already been published (*Monthly Notices, R.A.S.*, lxxii. pp. 714, 715). At least three settings were made on each line and a mean value deduced. These measures converted into wave-lengths, and the estimated intensities of the lines on a scale 1–10, form the first and second columns of the annexed table. In the case of the 1913 photographs all the six plates were similarly measured independently, and no line was admitted into the final list that did not appear on at least two plates. Some of the lines were very faint and the settings difficult to make, but a satisfactory mean result has been obtained. The brightest band in all these plates was the nebular band 4363. The middle of this band, which extended over 20 Ångström units, was selected as the fiducial point with a wave-length 4360. This gave the reading 4341 for H_γ, which coincided with the wave-length of the more refrangible edge of the H_γ band in the 1912 photograph. The whole spectrum with the objective prism from w.l. 5100 to w.l. 3820 only covers 8 mm., but it has nevertheless



1. Spectrum of Nova Persei, 1901 Aug. 27. Mag. 6·6.
2. Spectrum of Nova Geminorum (2), 1913 March 11. Mag. 8·8.

been possible to measure 53 lines and maxima of bright bands. The advantage of a small instrument in cases of such bright-line or bright-band spectra is that it concentrates the light, whereas an instrument giving greater dispersion would spread the light over a much greater breadth of spectrum. The measures with their estimated intensities on a scale 1–10 are contained in the third and fourth columns of the table. A vinculum containing two or more measures signifies that these lines apparently constituted a band. Of course with such small resolving power two bands might easily coalesce into one. The fifth column of the table contains a list of nebular lines taken from the table in Scheiner's *Astronomical Spectroscopy* (Frost's translation), pp. 227, 228. In the sixth column is given a list of chromospheric lines from Dr. Dyson's table (*Memoirs, R.A.S.*, lvii., Appendix), and their corresponding chemical origins, so far as could be ascertained. The number of close concordances with the wave-lengths of the lines of the Nova in 1912 is very striking.

As a matter of curiosity, to test the accuracy of the wave-lengths deduced with the objective prism, the numbers were compared with the extensive lists of wave-lengths given in the paper "Observations of the Spectrum of Nova Geminorum No. 2" by Walter S. Adams and Arnold Kohlschutter (*Astrophysical Journal*, xxxvi. pp. 293-321), from the photographs obtained in 1912 with the Mount Wilson 60-inch reflector and single prism slit spectrograph. These measures are given to five significant figures. They were reduced to four, by taking the nearest ten, and in the case of two or three measures of the same line a mean value was obtained. The concordance, considering the great disparity in the size and features of the two instruments, is most satisfactory. Forty-seven lines occur in the part of the spectrum $\lambda 3883 - \lambda 5102$ covered by both sets of measures, and of these 38, or about 85 per cent., are represented within a few units, generally two or three, by the Mount Wilson figures. The Mount Wilson measures refer to bands containing both bright and dark lines. It is impossible to fit the dark-line spectrum as a whole to the wave-lengths from the Stonyhurst spectrograms. But the fit with bright lines or maxima of brightness in the complicated bands measured at Mount Wilson is a good one, for 24 lines, or 50 per cent., are concordant with the centres or bright maxima of the Mount Wilson bands, 10 lines, or 22 per cent., with the violet edges of bright bands, and 5 per cent with the red-side edges. There are only 5 lines concordant with dark lines, and 6 lines may be common to dark or bright lines in the Mount Wilson spectrograms. The seventh and eighth columns of the table contain these concordant measures from the Mount Wilson photographs and the description of the lines.

TABLE.

1912. April.		1913.		Origin.		Mount Wilson.
Wave-length.	Intensity.	Wave-length.	Intensity.	Nebulæ.	Chromosphere.	1912.
		3820 }	1			
		3836 }			Hη band.	
		3845 }		3848		
		3853 }		3854		
		3862 }	3	3868		
		3872 }		3870		
		3883 }	1			3883 Violet edge, bright band.
		3889 }			Hζ band.	3891 Centre, bright band.
		3908 }			3909	
		3929 }	4			3925 Violet edge, bright band.
		3939 }			Ca. K. band.	
		3950	1		3950 Y.	3951 Centre, dark line.
		3958 }	1			3955 Centre, bright band.
		3970 }	2		Hε band.	3971 Centre, bright band.
		3998 }	1	3998		
		4008 }	1			4008 Red edge, bright band.
		4028	1	4026	4028 Ti.	4035 Centre, faint bright band.
		4052	2		4054 Ti. V.	4052 Red edge, bright band.
		4060	1			4062 Violet edge, bright band.
4075	2	4070	3	4069		4070 Violet edge, bright band.
4081	3	4080	3		4081 Zr.	4081 Violet edge, strong dark line.
4086	3				4087 La.	
4097 }	4	4095 }	3			4096 Maximum in bright band.
4102 }	6	4102 }	3		Hδ band.	4102 Centre, bright band.
4109 }	6					
4111	1				4110 Nd.	
4132	2			4130	4133 Fe.	
4136	1				4136	
4141	2			4143	4144 He.	
4151	1				4151 Ti. Zr.	
4157	1			4154	4157	
4166	5			4167	4169 He.	
4177	1				4179 Fe.	
4181	1	4185	3		4181 C.	4184 Centre, bright band.
4192	1				4193	
4198	1				4198	
4207	4				4207	
4211	1				4211	
4217	2	4215	2		4216 Sr.	4217 Centre, faint bright band.
4221	2				4221	
4243	1			424		
4250	5				4250 Fe.	
4273	2	4273	2	4265	4274	4270 Centre, faint bright band.
4282	3	4286	1		4283	4285 Centre, faint bright band.
4289	4				4289	

TABLE—*continued.*

1912. April. Wave-length.	Intensity.	1913. Wave-length.	Intensity.	Nebulæ.	Origin. Chromosphere.	Mount Wilson. 1912.
4297	2				4297 Fe.	
...		4303	1		4303	4303 Centre, dark band.
4311					4310 Y.	4305 Centre, bright band.
4323	2				4323	
...		4328	1		4326 Fe.	4328 Centre, dark band.
4334	3					4330 Violet edge, bright band.
4341	8	4341	1		Hγ band.	4342 Centre, bright band.
4347	8	4350				4351 Centre, bright maximum.
4355	7		10	4364		
...		4360				4361 Centre, bright wing.
4373	1	4370				4373 Dark line, bright maximum.
4380	2	4375	4		4375 Y.	4376 Violet edge, bright band.
4390	4			4390	4388 He.	
4398	3					
4406	3	4407			4409 band.	4406 Violet edge, bright band.
4417			2		4418 Ti.	4425 Bright maximum.
4423	1	4427			4423 Y.	4429 Red edge, bright band.
4429	4				4427 Ti.	
4436	3				4438 He band.	
4444	5				4444 Ti.	
4452	4	4453	1			4452 Centre, dark band.
...		4477	1	4473	4472 He band.	4473 Centre, bright band.
4489	4				4489 Fe.	4478 Dark line.
...		4497	1		4497 Cr.	4496 Dark line; violet edge, bright band.
4506	2				4508 Fe.	
4521	4				4520 Fe.	
4528	4				4529 Fe.	
4543	4				4542 Fe.	
4604	3	4600	3			4601 Dark line, centre bright band.
4615	4			4610		
4630	10	4629	3		4630 Fe.	4627 Bright maximum.
4650	10	4643	3	4637		4645 Centre, bright maximum, dark line.
4660	3			4663		
4671	2	4670	3			4673 Centre, bright band.
4680	2			4687		
...		4695	2			4697 Bright maximum.
4716	1				4715	
4737	1				4737 Fe.	
4747	5			4744	4746	
4779	1				4778	
4798	1				4798	
4834	3	4825			4824 Cr.	4825 Dark line.
4843	2		2		4836 Cr.	
4852	10	4849				4849 Violet edge, bright band.
4862	10	4862		4862	Hβ band.	4863 Centre whole bright band.
4875	10					
4886	6					

TABLE—*continued.*

1912. April.		1913.		Origin.		Mount Wilson.
Wave-length.	Intensity.	Wave-length.	Intensity.	Nebulæ.	Chromosphere.	1912.
4892	4					
4903	4					
4921	4				4922 He band.	
...		4960	1	4959		4959 Centre, bright band.
4988	2	4975	4			4975 Red edge, bright band.
4990	2					
5003	2	5004	4	5007		5007 Centre, bright band.
5015	2				5016 He.	
5041	3					
5050	2	5055	1		5048 He.	5049 Centre, faint bright band.
5058	4	5060				5060 Red edge, bright band.
5071	3					
5083	3					
...	1	5102			5100	
5113	2					
5124	2					
5136	3					
5153	3				5154 Ti.	
5176	5				5173 Mg. b.	
5188	5				5184 Mg. b.	
5205	5				5206 Cr.	
5224						
5484	7				5483	
5503	6				5503 Fe.	
5600	3				5603	
5612	2				5616 Fe.	
5629	3					
5654						
5664	6				5663 Y.	
5681						
5694						
5713	2					
5737	5			5752		
5763	5					
5788	5					
5805	2					
5830	6					

Some visual observations of the spectrum of the star were also made with a McClean eye-piece spectroscope on the 15-inch equatorial. They add but little to the photographs except that H_α was very bright on 1912 April 19, April 22. On May 23 the star appeared very red in a low power ordinary eye-piece on the same instrument.

The Spectrum of 1912 *April.*—The bright-line spectrum consisted of a number of lines and of bands. Of the latter the

brightest were the nebular band 4637, which had gradually increased in intensity during the month until it equalled H_β at the end of the month, the bands H_β and, of less intensity, H_γ and H_δ. The helium spectrum was also most probably present, the lines 4144, 4169, 4388, 4438, 4922, and 5016 being represented by lines in the star 4141, 4166, a band 4380–90, 4436, 4921, and 5015. If columns 1 and 6 of the table be compared, it will be evident that the line spectrum was a chromospheric spectrum in which enhanced metallic lines were prominent, chiefly of titanium and iron, with evidences of yttrium, zirconium, strontium, chromium, and vanadium. With regard to titanium, the lines 4390, 4398, 4417, 4444 are represented in the dark-line spectrum of Newall and Stratton in the early stage of the life-history of the star (*Monthly Notices, R.A.S.*, lxxiii. pp. 380–382), by 4392·2, 4399·9, 4417·9, and 4444·0. Lockyer gives enhanced lines of titanium at 4417·9 and 4444·0. Again, with regard to iron, there are two enhanced lines in Lockyer's lists at 4508·5 and 4522·8, with intermediate lines at 4515·5 and 4520·4. Newall and Stratton give a band of four lines with extremities 4508·5 and 4522·8. The Mount Wilson photographs too show a dark line at 4508·1, 4508·0, and 4506·8 in three sets of photographs, and another dark line in two sets of photographs at 4522·8 and 4522·1. The Stonyhurst plate of April 24 gives bright lines at 4506 and 4521, the latter the violet edge of a bright band. Those observations would apparently contravene those of Professor Küstner (*Astronomische Nachrichten*, No. 4654), who would exclude enhanced titanium and iron from the dark-line spectrum of the star. The conclusion I would draw from a comparison of the Stonyhurst plate with the observations at Cambridge and Mount Wilson, is that the dark-line enhanced spectrum of March became a bright-line spectrum in April, though some dark lines persisted to the end of May.

The Spectrum of 1913 *March, April.*—The most prominent feature in this spectrum is the series of bright nebular bands in the following order of brightness: 4363, much in excess of the others, 5007, 4069, 4637. The whole hydrogen series from H_β to H_η was present, H_β and H_γ very faint, H_δ the brightest of all. H_δ extended over 7, H_ϵ over 12, H_ζ over 6, and H_η, which was difficult to measure, over 16 units of wave-length. The K band of calcium is present, and probably the H band conjoined with that of H_ϵ. But besides the nebular and hydrogen spectrum, there are certain other bright lines. The three following are interesting, especially as two are enhanced lines of titanium, the third is a well-marked enhanced line of strontium:—

Lockyer.	Cambridge, 1912 March, Dark Lines.	Mount Wilson, 1912 April 22, Dark Lines.	Stonyhurst, Bright Lines, 1912 April 24.	1913 April.
4215·7 Sr.	4215·7	4215·6	4217	4215
4374·9 Ti.	4374·9	...	4373	4375
4417·9 Ti.	4417·9	4418·2	4417	4417

The wave-length 4417 of 1913 is the mean of 4407, and 4427 the edges of a bright band; 4376·0 is a bright line in the Mount Wilson spectra of March. In addition, as has been already mentioned, about 50 per cent. of the measures are concordant with the centres of bright bands measured in 1912 March to May at Mount Wilson. The spectrum is a spectrum of remnants, with a great relative increase in the brightness of the nebular bands, and an inversion of brightness between the bands 4637 and 4364 in the interval 1912 April and 1913 April. But seeing that the nebular bands and the hydrogen bands and some of the enhanced chromospheric metallic lines have persisted as present in the spectrum in spite of fluctuations in brilliancy from 1912 March 22 to 1913 April, there is a permanent character about the spectrum which would suggest that the dictum "new stars change into nebulæ" is not justified. If this is so, it is obvious that the simplest explanation of the origin of a new star is an outburst, a solar chromospheric storm on a greatly enhanced scale, in a nebulous star.

Stonyhurst College Observatory:
1913 *June* 10.

Note on Variable Stars of Cluster Type.

By H. C. Plummer.

1. The variable stars which have been found in clusters belong mainly to a certain well-marked type, the most characteristic feature of which is the extremely short period of variation. Indeed, when the difficulties of an exact classification are considered, it seems not impossible that it may be well to group together provisionally all those stars which complete a regular cycle of change within a period less than one day. A rich store of information is contained in the beautiful memoir of Professor S. I. Bailey on the ω Centauri cluster.* But stars of this type are not confined to clusters, nor are they to be found in any abundance in all clusters. The A.G. catalogue of variables (1912) gives a list of fourteen such objects unconnected with clusters, and the shortness of the list may be fairly attributed to a purely arbitrary system of classification. Some of these stars have been carefully observed, and it is the object of this note to make some comparative study of the results.

2. The stars of this class which seem to have been best observed, five in number, are given in Table I.

* *Harvard Annals*, xxxviii. (1902).

TABLE I.

No.	Stars.	Period. d	Method.	Authority.
1	ST Virginis	0·410608	vis.	Guthnick *
2*a*	ST Ophiuchi	·450363	,,	,,
2*b*	,,		,,	Shapley †
3	S Aræ	·451892	,,	Roberts ‡
4*a*	SW Draconis	·569666	,,	Sperra §
4*b*	,,		phot.	Dunsink ‖
5*a*	SU Draconis	·660419	vis.	Sperra §
5*b*	,,		phot.	Dunsink ‖

Thus one of the five stars has been observed independently by two sets of visual observers, while two others have been observed both visually and photographically.

3. Bailey has distinguished three sub-classes among the ω Centauri variables. The want of homogeneity in the above material makes it practically impossible to make any useful comparison of the light ranges. But with regard to period, the ω Centauri results may be expressed thus :—

TABLE II.

Sub-Class.	No.	Mean Period. d	Standard Dev. d	Range. d d
a	33	0·58	0·047	0·49–0·66
b	15	·75	·063	0·67–0·90
c	29	·39	·064	0·30–0·62

On the basis of period alone, therefore, we should assign the stars 1–3 to sub-class *c*, and the stars 4 and 5 to sub-class *a*. But the other characters of the curves do not seem to agree so well with this division.

4. The eight curves of the five stars have been submitted to periodic analysis, which gives the magnitude in the form

$$m = m_0 + \Sigma r_n \sin\,(n\theta + \gamma_n),$$

where

$$\mathrm{P}\theta = 2\pi(t - t_0),$$

P being the period and t_0 the time of maximum. The resulting phases γ_n are tabulated in Table IV., and the coefficients r_n in Table V.

* *A. N.*, 4284. † *Laws Bull.*, 17.
‡ *Ap. J.*, xxxiii. p. 208. § *A.N.*, 4407.
‖ *M.N.*, lxxiii. pp. 166 and 440.

TABLE IV.

	Star 1.	2		3	4		5	
		a	*b*		*a*	*b*	*a*	*b*
n								
1	238°	231°	227°	230°	237°	231°	236°	240°
2	238	230	226	229	253	242	235	·272
3	258	243	251	236	299	307	277	302
4	269	255	260	255	281	302	331	345
5	286	263	281	266	296	310	288	166
6	302	280	298	284	334	326	4	12
7	321	296	315	312	354	207	60	268
8	330	331	333	331	(81)	221	(315)	244
9	344	357	350	351	198	292	51	237
10	(0)	11	25	11	207	96	139	237
11	(304)	33	47	61	(258)	158	333	146

TABLE V.

	Star 1.	2		3	4		5	
		a	*b*		*a*	*b*	*a*	*b*
n								
1	0·41	0·45	0·63	0·47	0·30	0·36	0·28	0·34
2	·19	·21	·22	·24	·12	·18	·12	·19
3	·13	·16	·20	·16	·09	·07	·07	·15
4	·08	·11	·11	·10	·03	·05	·05	·05
5	·06	·09	·10	·07	·02	·04	·02	·03
6	·03	·06	·06	·05	·01	·02	·03	·06
7	·02	·04	·06	·04	·01	·01	·01	·03
8	·01	·03	·04	·03	...	·03	...	·02
9	·01	·03	·03	·02	·01	·02	·01	·02
10	...	·03	·02	·01	·01	·02	·01	·01
11	...	·02	·01	·01	...	·03	·01	·01

5. Before any attempt is made to draw a deduction from these results of the analysis, it is necessary to consider them with the greatest caution. The coefficients of the higher terms are small, and the accuracy of the corresponding phases naturally tends to be low. Besides this, the general character of the curve is represented by the whole of the terms, and it is unsafe to pick out a single term or group of terms as corresponding to a particular feature of the curve. An opportunity may be taken here to correct an unfortunate slip in the note added at the end of the paper on SU Draconis (p. 173), and the correction supplies a good illustration of this point. As already indicated for the star 5*b*, the fifth, sixth and seventh terms can be combined thus:—

$$\cdot 06 \sin (6\theta + 12^\circ) - \cdot 06 \sin (6\theta + 37^\circ) \cos (\theta + 51^\circ).$$

This expresses a wave with one-sixth of the whole period, which reaches a maximum amplitude of ·12 when $\theta = 129^\circ$, and dies

away in the opposite half of the period. It corresponds with a visible feature of the photographic light curve. But if we group together the corresponding terms in the expression of the visual light curve, we obtain :—

$$\cdot 028 \sin (6\theta + 4^\circ) + \cdot 028 \sin (6\theta - 6^\circ) \cos (\theta + 66^\circ).$$

The correction has reversed the connecting sign from − to +, and the effect is to place the maximum of the secondary wave at $\theta = 294^\circ$. This is quite near the minimum of the visual light curve and no longer corresponds to any obvious property of either curve. The point of resemblance which is left to the two curves in this respect is now confined to the sixth components alone. This fact must modify the remarks based on a supposed more intimate relation between the two curves. But it is fair to remember that the

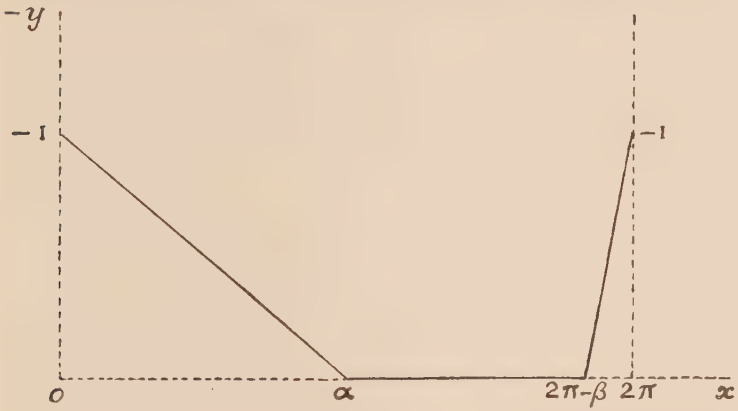


Fig. 1.

coefficients which stand in opposition are exceedingly small, and that a real difference between visual and photographic results is quite possible.

6. The need for caution in the interpretation of the results of harmonic analysis goes much deeper, however, and may be illustrated by a simple artificial example. Let us consider the harmonic representation of the periodic function of which the graph in one period is shown in fig. 1.

The graph consists of three straight lines joining successively the points $(0, -1)$, $(\alpha, 0)$, $(2\pi - \beta, 0)$, $(2\pi, -1)$. It is the crudest possible representation of a limiting form of cluster-type magnitude curve. If we seek to represent the function by a Fourier series, we obtain without difficulty

$$y = -\frac{\alpha+\beta}{4\pi} - \frac{2}{\pi}\sum \frac{\cos nx}{n^2}\left(\frac{1}{\beta}\sin^2 \tfrac{1}{2}n\beta + \frac{1}{\alpha}\sin^2 \tfrac{1}{2}n\alpha\right)$$
$$- \frac{1}{\pi}\sum \frac{\sin nx}{n^2}\left(\frac{1}{\beta}\sin n\beta - \frac{1}{\alpha}\sin n\alpha\right)$$
$$= -(\alpha+\beta)/4\pi + \Sigma r_n \sin (nx + \gamma_n)$$

where

$$r_n = \frac{2}{\pi n^2}\left\{\frac{1}{\beta^2}\sin^2\tfrac{1}{2}n\beta - \frac{2}{\alpha\beta}\sin\tfrac{1}{2}n\alpha\sin\tfrac{1}{2}n\beta\cos\tfrac{1}{2}n(\alpha+\beta) + \frac{1}{\alpha^2}\sin^2\tfrac{1}{2}n\alpha\right\}^{\frac{1}{2}}$$

and

$$\tan\gamma_n = \frac{\alpha(1-\cos n\beta)+\beta(1-\cos n\alpha)}{\alpha\sin n\beta - \beta\sin n\alpha}, \qquad (\pi<\gamma_n<2\pi).$$

The phases of all the terms, reckoned from maximum, always lie in the third and fourth quadrants.

7. A numerical example will illustrate the nature of the series. We choose the case in which $\alpha=\pi$, $\beta=\pi/6$. With these values the coefficients and phases are as follows:—

n.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
r_n	·416	·152	·114	·066	·059	·034	·030	·017	·014	·006	·004
γ_n	223°	210°	233°	240°	257°	270°	283°	300°	307°	330°	317°

This bears a very close resemblance with the series found to represent the magnitude curve of ST Virginis. Again, if we calculate the phase of the first term for a range of values of α and β, we obtain the results:

α \ β.	30°.	45°.	60°.
150°	232°	237°	242°
180	223	228	233
210	215	220	226

From this we must infer that the value of γ_1 changes but little when there is a considerable change in the shape of our artificial function.

8. The inferences to be drawn from the consideration of the foregoing illustration seem very instructive in a practical sense and full of warning against hasty deductions from the results of harmonic analysis. The star ST Ophiuchi was observed by two sets of observers ($2a$ and $2b$). Between their results there were several discrepancies of detail—range of magnitude, terminal points of the stationary phase, sinuosities in the curve. These divergences are not markedly apparent in the harmonic representation, and the essential agreement is beautifully emphasised. Indeed, the only serious difference appears in the amplitudes of the first term. At the same time, the star S Aræ (3) is shown to be represented by a series which is practically as close to that of ST Ophiuchi as the two determinations of the latter are to one another. Nor is the series for ST Virginis very different. That the nature of the physical process operating in each of the three stars is identical seems beyond doubt. But on the nature of the process itself the analysis throws absolutely no light. We have seen that the mere *form* of the function is paramount in determining the nature of the series, and the property which is really brought out by the analysis is the slow and approximately straight descent of the

curve from maximum, the stationary phase at minimum, and the rapid and approximately straight ascent to maximum again. But this is no additional information, and it seems clear that a fairly subtle interpretation of the harmonic series would be necessary in order to make any more precise inference.

9. The path of advance along this line seems therefore, for the moment at least, obscure. We are compelled to fall back on a general consideration of the magnitude curves as represented graphically. But here again the difficulties of interpretation are not slight. We have seen reason to attribute the apparent resemblance of the harmonic series for the stars 1–3 to the assumed existence of a constant phase at minimum. On the other hand, the comparative irregularity shown by the stars 4 and 5 is undoubtedly connected with the absence of this characteristic feature. The all-important question for this class of variables now is whether this supposed constant phase is a reality, and it is naturally a most difficult one. To detect a small variation is difficult enough; to establish beyond doubt the absence of variation is practically impossible. The observers of ST Virginis and ST Ophiuchi have accepted the existence of the stationary phase without explicit critical examination. But in the case of S Aræ Dr. Roberts has made a very large number of observations at this phase. Not only does he conclude therefrom that the light at minimum is absolutely stationary, but he has based on this conclusion an elaborate theory of the star as an eclipse variable arising from a binary system in which one component is constant and the other itself variable. Now if we plot his mean magnitudes from Table III. (*loc. cit.*, p. 208), and draw a curve through them as faithfully as possible, we find the clearest indication that, so far from the light remaining constant at minimum, it undergoes a distinct fluctuation within very narrow limits, and it is a pure assumption that the change is not absolutely continuous throughout the curve. And the only effective way of supporting the alternative conclusion is to insist that too much weight is laid here on the observational evidence. That is always possible, but if the only evidence is impugned the question remains completely open.

10. It appears then that the stationary phase may be only an apparent discontinuity in the curve. If this be so, there is no reason to draw any clear distinction of class between the three stars ST Virginis, ST Ophiuchi and S Aræ on the one hand, and SW and SU Draconis on the other. But now we may notice that SW Draconis does possess an approach to a stationary phase, only it *precedes* the minimum. And SU Draconis apparently possesses no stationary phase at all. The approach to constancy may, as it seems, occur at any part of the curve (except perhaps on the rapid rise), or again it may not occur at all. It seems even conceivable that it might occur at maximum and lead to an erroneous classification as an Algol star. And in any case, if the association of the fourteen stars in the A.G. list is to be preserved, we must abolish altogether or limit the use of the misleading

term Antalgol. It cannot be applied to SU Draconis with any propriety.

11. The main inference from considerations of this kind is that there is no present necessity to accept an eclipse theory for any star of this class, although obviously a pair of eclipsing stars, each varying according to arbitrary laws, is capable of explaining any magnitude curve. But an *a priori* objection may possibly be taken to this view. We know that variables of the δ Cephei type, when submitted to the test, have invariably proved to be spectroscopic binaries. It would be entirely reasonable to extend this fact to the cluster type variables. The greater complexity of the curves in the latter class may be due to the natural struggle between forced and free oscillations associated with the shorter periods, much in the same way as terrestrial tides would probably be much simpler if the earth's rotation were slower. The photographic results for the cluster type variables show a greater range of magnitude than the visual, and this suggests an analogy with the fact noted by Albrecht for the δ Cephei type, that the maximum intensity of the continuous spectrum advances towards the violet end as the star brightens. The average range of light variation is much the same in both classes, and if further we accept continuous change as a general condition, there remains little beyond period to distinguish the two classes. Here, then, arises the difficulty in rejecting the possibility of eclipse effects altogether. If the light variation is associated with an orbital motion completed in a few hours, the two bodies must be extremely close and, merely in accordance with probability, the conditions necessary for an eclipse will be frequently satisfied. The pertinence of this consideration is illustrated by the discovery of the variability of β Aurigæ by Mr. Stebbins. It is not lightly to be set aside. But it depends for its validity on the assumption that stars of cluster type are binary systems. The type unfortunately contains no bright representative star whose radial velocity could be measured accurately, if at all. If in any way feasible, it is greatly to be desired that an attempt should be made to do this.* In the meantime, a condition of quite a different kind, such as a prominence effect on a large scale, does not seem altogether excluded.

12. Questions of classification and nomenclature have naturally been alluded to, and although it must be felt that this subject as regards variable stars is in a thoroughly unsatisfactory state, yet it would be premature at the present time to attempt any general revision. It has also been pointed out in one instance that by taking an observer's own mean magnitudes without that process of smoothing which, if one may be pardoned for saying so, is the besetting sin of variable star observers, a conclusion was drawn exactly contrary to that formed by the author himself. Both of

* It has been done. The star RR Lyræ (period $0^d{\cdot}566826$) has been studied by Dr. Kiess and found to be a spectroscopic binary. *Publ. Astr. Soc. Pacific*, XXV. p. 121 (1913 June).

these points are illustrated by the star RV Ursæ Majoris, studied by so well known an observer as M. Luizet.* The period is $0^d.468116$, which in itself suggests a cluster type variable. M. Luizet has assigned it to the δ Cephei class. But if we plot the author's mean magnitudes and draw the curve fairly through them without smoothing, we recognise at once the affinity to the curves of SU and SW Draconis. The curve is a particularly interesting one, showing three marked secondary oscillations each in one-sixth of the whole period in the neighbourhood of the minimum. This fact, which is clear enough in the graphical form, drops out of evidence in the harmonic representation, which runs thus:

n.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
r_n	0·35	·16	·08	·04	·04	·02	·01	·02	...	·01	·01
γ_n	254°	273°	284°	306°	322°	240°	279°	338°	(292°)	186°	326°

The amplitudes r_n, which are deduced on the assumption that the observer's "step" is $0^m.1$ (range $8^{st}.9 = 0^m.9$), correspond, it will be noticed, very closely with those given for star 4*b* in Table V. It may well seem presumptuous to criticise experienced observers of variable stars, but the criticism is based on the belief that in many cases the observers attach too little weight to the accuracy of their own work. At the same time, there can be no harm in throwing doubt on what seems to be too prevalent a notion, that smooth curves are in the nature of things.†

13. Another example may be referred to which has presented itself very recently,‡ in fact after what precedes had been written. Mr. Shapley has discussed the star S Antliæ as an ellipsoidal variable. The physical nature of the star is beside the purpose of the present paper. But we may notice that the period is $0^d.32417$. This at once suggests a variable of cluster type. We are then led to suspect irregularities in the form of the magnitude curve. A glance at the curve provided shows at once that they are present. Unfortunately, the whole range of variation is very small, so that it is difficult to lay great weight on small deviations from the curve of reference. But they exist, and they leave the agreement between theory and observation far from satisfactory. The range is said to be $0^m.35$, but would appear to be much more nearly $0^m.40$. So far as proper classification goes the case seems particularly easy, as will be seen at once if we quote Bailey's description of sub-class *c*.§

"Sub-class *c*. Period and light curve perhaps uniform. Light appears to be always changing, and with moderate rapidity. Increase of light generally somewhat more rapid than the decrease, but in a few cases it appears to be of only equal or of less rapidity" (observers of S Antliæ differ as to which is the case). "In this cluster the range is generally somewhat more than half a magnitude,

* *A.N.*, 4292.

† This doubt is confirmed by M. Luizet's own most recent work on Y Ophiuchi (*Bull. Astr.*, xxx. p. 275) and RS Orionis (*A.N.*, 4662).

‡ *A.N.*, 4653.

§ *Harvard Annals*, xxxviii. p. 132.

and the period from eight to ten hours, although these limits are sometimes exceeded."

From the obvious appropriateness of this description to S Antliæ, two deductions may fairly be drawn. The distinction between stars of δ Cephei type and the cluster variables either ought, or ought not, to be maintained. If it is maintained, then the line of separation ought to be drawn with a great deal more care than has hitherto been the practice. In the second place, the utility of applying a definite theory to a selected star seems very doubtful unless that theory can be extended to the whole class to which the star belongs, especially when the fit between the resulting curve and the observations is so indifferent as in the case mentioned.

14. It has been pointed out by more than one correspondent that the nebula found in the neighbourhood of SW Draconis is N.G.C. 4236, a fact overlooked here by an unfortunate mischance. The description runs:—

*v*F, *e*L, *m*E 160° ±, *vgb*M.

I am indebted to Professor A. A. Nijland for the following note:—

"The nebula was seen by me (visually) for the first time on March 11; on that date I used a low-power negative eyeglass that yields an extensive field. The nebula could be traced, in the direction given on your chart, to 5′ N. of star 2 and to the centre of the triangle of stars preceding *a* and 1. . . . The description . . . corresponds exactly with what I can see." The instrument used was a 10-inch refractor ($a = 26$ cm., $f = 319$ cm.) with the power 83.

To Professor Barnard I am indebted for a photograph of the region taken very kindly at my request with the Bruce 10-inch of the Yerkes Observatory on 1913 April 28, exposure $4^h\ 22^m$. This shows the same form as was obtained here with the 15-inch reflector and the 6-inch doublet. The region of SU Draconis falls on the same plate, but towards the edge, where the definition is inferior. No nebulosity is apparent in this region, and this is confirmed by a three hours' exposure made on April 3 by Mr. Pease with the 60-inch reflector on Mount Wilson, for a contact positive of which my thanks are due to Mr. Adams. We may safely infer that the immediate neighbourhood of SU Draconis is free from nebulosity. The evidence for the variability of N.G.C. 4236 is of the slightest, but it is not absolutely impossible if we suppose the luminosity of the nebula excited in some manner by a star itself variable, as suggested by Mr. Reynolds.*

Dunsink Observatory:
1913 *May* 29.

* *M.N.*, lxxii. p. 553.

Note on the Orbit of ζ Geminorum. By H. C. Plummer.

1. It is well known that ζ Geminorum is not only a variable star but that its motion in the line of sight is also variable in the same period as its light ($10^d \cdot 154$). This motion has been carefully discussed by Campbell (*Ap. J.*, xiii. p. 90), with the interesting result that the radial velocity of the star cannot be reconciled with motion in a purely Keplerian ellipse. In this it is not unique, for other stars have shown a similar deviation. In other cases, however, the observations have generally been made with a one-prism spectrograph, and it might be suggested that the effect was due to the superposition of the fainter spectrum of the companion and a consequent disturbance of the measured "blends." But an explanation of this kind is not so readily available in the case of ζ Geminorum, because the spectrograms were obtained with apparatus of higher dispersion, and yet no evidence of a secondary spectrum was noticed. For this reason ζ Geminorum seems deserving of special attention, and, although it is not hoped to find an explanation of the anomaly, an attempt may be made to define its nature a little more exactly.

2. It has been usual, in discussing the orbits of spectroscopic binaries, to employ a graphical method. In this way a curve is obtained which corresponds to elliptic motion, and which fits the observations as closely as possible. But when a discrepancy is apparent this process is not well adapted to bring out its true character. For this purpose the analytical method based on a harmonic analysis of the curve is better suited. This has also been suggested by Russell,* who refers to an investigation of ζ Geminorum, which has not, I think, been published. In the meantime there can be no harm in discussing the question on these lines, even if the result is only to call attention to the serious difficulties of the particular problem.

3. If we read off 24 equidistant ordinates of the velocity curve, the harmonic analysis is very easily performed. If the velocity is expressed in the form

$$V = V_0 + \Sigma r_j \sin(j\mu t + \gamma_j),$$

the constants of the series which we obtain are as follows:—

j	1	2	3	4	5	6	7	8	9	10	11
r_j	12·6	2·9	1·3	0·2	1·3	0·1	0·8	0·2	0·5	0·1	0·3
γ_j	111°	167°	107°	135°	93°	236°	272°	248°	91°	225°	280°

with $V_0 = +7 \cdot 0$ km. per sec. for the velocity of the system. The epoch is the time of visual minimum, and μ is $360^\circ/P$ where P is the period. The 24 ordinates run thus: +21·2, +17·4, +11·6, +7·0, +4·6, +3·3, +2·1, +1·7, +0·7, −0·3, −2·4, −4·6, −6·4, −5·1, −2·0, +0·7, +3·8, +7·1, +10·6, +14·0, +17·0, +19·5, +21·9, +23·7 (km. per sec.). The feature which we

* *Ap. J.*, xv. p. 260.

notice at once is the survival of the odd amplitudes r_j and the association with them of phases near 90° or 270°.

4. If the harmonic series found above were really consistent with elliptic motion we should have the series of equations

$$r_j \sin(j\mu T + \gamma_j) = 2K \cot\phi \cos\phi \cos\omega J_j(je) \quad . \quad . \quad (1)$$

$$-r_j \cos(j\mu T + \gamma_j) = 2K \cos\phi \sin\omega J_j'(je) \quad . \quad . \quad . \quad (2)$$

completely satisfied.* Here T is the time of periastron passage, K is a constant velocity, ω is the longitude of periastron, e is the eccentricity, $\sin\phi = e$, and J_j denotes a Bessel's coefficient of order j, J_j', its derivative. We shall now find the four constants K, T, ω, and e so as to reproduce the first two terms of the series, and we give the steps in some detail because the method does not seem to have been used so much as it deserves to be.

5. If we write

$$K_1 = K\cos^2\phi\cos\omega, \qquad K_2 = K\cos\phi\sin\omega,$$

the first four equations ($j = 1, 2$) become

$$r_1 \sin(\mu T + \gamma_1) = K_1\left(1 - \frac{e^2}{8} + \frac{e^4}{192} - \ldots\right) \quad . \quad (3)$$

$$-r_1 \cos(\mu T + \gamma_1) = K_2\left(1 - \frac{3e^2}{8} + \frac{5e^4}{192}\right) - \ldots \quad . \quad (4)$$

$$r_2 \sin(2\mu T + \gamma_2) = K_1 e\left(1 - \frac{e^2}{3} + \frac{e^4}{24} - \ldots\right) \quad . \quad (5)$$

$$-r_2 \cos(2\mu T + \gamma_2) = K_2 e\left(1 - \frac{2e^2}{3} + \frac{e^4}{8} - \ldots\right) \quad . \quad (6)$$

Hence

$$\frac{r_2}{r_1} \cdot \frac{\sin(2\mu T + \gamma_2)}{\sin(\mu T + \gamma_1)} = e\left(1 - \frac{5e^2}{24} + \frac{e^4}{96} \ldots\right) \quad . \quad . \quad (7)$$

$$\frac{r_2}{r_1} \cdot \frac{\cos(2\mu T + \gamma_2)}{\cos(\mu T + \gamma_1)} = e\left(1 - \frac{7e^2}{24} - \frac{e^4}{96} \ldots\right) \quad . \quad . \quad (8)$$

and, by subtraction and addition,

$$\frac{r_2}{r_1} \cdot \frac{\sin(\mu T + \gamma_2 - \gamma_1)}{\sin 2(\mu T + \gamma_1)} = \frac{e^3}{24} + \frac{e^5}{96} \ldots \quad . \quad . \quad (9)$$

$$\frac{r_2}{r_1} \cdot \frac{\sin(3\mu T + \gamma_2 + \gamma_1)}{\sin 2(\mu T + \gamma_1)} = e\left(1 - \frac{e^2}{4} \ldots\right) \quad . \quad . \quad (10)$$

Now r_2/r_1 is the first approximation to e. Hence if e is small $\left(< \frac{1}{2} \text{ say}\right)$ (9) shows that $\mu T + \gamma_2 - \gamma_1$ is a very small angle indeed, so that we may write

$$\mu T = \gamma_1 - \gamma_2 \quad . \quad . \quad . \quad . \quad (11)$$

as an excellent approximation from the beginning. Then (10) becomes simply

$$e - \tfrac{1}{4}e^3 = r_2/r_1 \quad . \quad . \quad . \quad . \quad (12)$$

* *Cf. Ap. J.*, xxviii. p. 219.

with the neglect of only a small term involving the seventh power of e on the left. These simple equations, (11) and (12), will often suffice to give T and e directly without further appeal to (9) and (10), by which, in any case, a final adjustment can be made if necessary without any difficulty. In the present case (12) gives $e = \cdot 233$ and (11) gives $\mu T = 56^{\circ}$, a value only altered by the negligible amount $7'$ when a second approximation is deduced from (9).

6. If we return now to equations (3) and (4), and use the values just found for e and μT, we obtain

$$K_1 = 10\cdot 4, \qquad K_2 = -7\cdot 3,$$

which give

$$\omega = -34^{\circ}\cdot 5, \qquad K = 13\cdot 3 \text{ km. per sec.}$$

The elements of the orbit can now be set down in a collected form with Campbell's values in brackets for comparison:

$$\begin{aligned}
&\text{Velocity of system} = +7\cdot 0 \ (6\cdot 8)\\
&A = K(1 + e\cos\omega) = 15\cdot 9 \ (15\cdot 7)\\
&B = K(1 - e\cos\omega) = 10\cdot 7 \ (10\cdot 7)\\
&e = 0\cdot 233 \ (0\cdot 22)\\
&\omega = 325^{\circ}\cdot 5 \ (333^{\circ})\\
&T = -1^{d}\cdot 58 \ (1\cdot 313)\\
&a \sin i = 1{,}806{,}000 \text{ km. } (1{,}797{,}800).
\end{aligned}$$

Campbell gives T with the opposite sign, but there seems to be no disagreement as to the fact that the periastron passage *precedes* the light minimum by this interval. The agreement, on the whole, is very fair, as we should expect. It may be remarked that the analytical method followed here is extremely expeditious, and has every claim to at least equal consideration with any graphical method.

7. This is true in the normal case. But in an abnormal case like the present there is a special advantage in that the orbit found satisfies definite conditions. That is not to say that it is in any special sense the right orbit, but simply that any deduction based upon it is more readily open to the control of critical interpretation. In rather similar cases it has been supposed that the period of the inequality is either equal to the whole period of the star, or to one-half or one-third of this period. But these are consequences of applying the principle of "closest fit" in the graphical sense, and it must be very difficult to be sure of the reality of these periods in preference to others. Here we have thrown the terms having the whole period and the half period definitely into the elliptic motion and thereby satisfied the two most important terms. It is by no means certain that the whole of the first term, for example, should have been disposed of in this way, but at all events what has been done is quite clear.

8. We proceed to compare the series found to represent the velocity projected on the line of sight with that which results from the elliptic orbit. We have

$$2e^{-1}J_3(3e) = \frac{9}{8}e^2\left(1 - \frac{9}{16}e^2 + \frac{81}{640}e^4 - \ldots\right) = \cdot059$$

$$2J'_3(3e) = \frac{9}{8}e^2\left(1 - \frac{15}{16}e^2 + \frac{189}{640}e^4 - \ldots\right) = \cdot058$$

$$2e^{-1}J_4(4e) = \frac{4}{3}e^3\left(1 - \frac{4}{5}e^2 + \frac{4}{15}e^4 - \ldots\right) = \cdot016$$

$$2J'_4(4e) = \frac{4}{3}e^3\left(1 - \frac{6}{5}e^2 + \frac{8}{15}e^4 - \ldots\right) = \cdot016$$

and it is unnecessary to go further, because the next coefficients of K_1 and K_2 fall to ·004 and produce a term less than 0·1 km. in amplitude. Accordingly, the elliptic motion can be sufficiently represented by the series

$$V_e = 12\cdot6 \sin(\mu t + 111^\circ) + 2\cdot9 \sin(2\mu t + 168^\circ) + 0\cdot7 \sin(3\mu t + 223^\circ) + 0\cdot2 \sin(4\mu t + 279^\circ).$$

9. The third and fourth terms in V_e may be written

$$-0\cdot5 \sin 3\mu t - 0\cdot5 \cos 3\mu t + 0\cdot0 \sin 4\mu t - 0\cdot2 \cos 4\mu t,$$

while the corresponding terms in V are

$$-0\cdot4 \sin 3\mu t + 1\cdot2 \cos 3\mu t - 0\cdot1 \sin 4\mu t + 0\cdot1 \cos 4\mu t.$$

Thus the residual terms in $(V - V_e)$ are

$$+0\cdot1 \sin 3\mu\tau + 1\cdot7 \cos 3\mu t - 0\cdot1 \sin 4\mu t + 0\cdot3 \cos 4\mu t$$

together with the subsequent terms given in § 3. And now, if we confine our attention to those terms which seem to be the most important, we may write

$$V - V_e = +1\cdot7 \cos 3\mu t + 1\cdot3 \cos 5\mu t - 0\cdot8 \cos 7\mu t + 0\cdot5 \cos 9\mu t - 0\cdot3 \cos 11\mu t.$$

Here we have omitted the term $0\cdot3 \cos 4\mu t$, but no other term with an amplitude exceeding 0·2. The last term has been retained, although small and necessarily uncertain, because it falls into line with what seems to be the characteristic property of this residual series, that it consists principally of cosines of the odd multiples of μt. It seems distinctly possible that there ought to be a term involving $\cos \mu t$, but, as already fully explained, this has been absorbed into the elliptic motion. Yet this may fairly be said: unless the eccentricity of the orbit is considerably greater than we have supposed, the elliptic series will converge rapidly and the general character of the later outstanding terms will not be affected. If we denote $V - V_e$ by $f(\mu t)$, this function has the same symmetrical properties as $\cos \mu t$, that is to say,

$$f(\mu t) = -f(\pi - \mu t) = -f(\pi + \mu t) = f(2\pi - \mu t).$$

The importance of this remark lies in the fact that the adopted epoch is the time of light minimum and the light curve is also very nearly symmetrical. This in itself suggests an intimate connection between the inequalities in the observed velocity and

the light variation of the star, though of course it does not explain either the one or the other.

10. As regards an explanation, the present discussion does seem to point to one definite inference. Granted the main periodicity ruling the inequalities, it is difficult to see how they can be reconciled with a triple system. Such a system has often been suggested with no regard to the complexity of the problem of three bodies, and subsidiary orbits with commensurable periods have been introduced as lightly as the ancients invoked a fresh epicycle to meet each recurring difficulty. For ζ Geminorum Campbell suggests the possibility of a secondary orbit described in one-third of the main period, and dismisses the idea at once on the ground of instability. A similar difficulty appears to be responsible for Russell's delay in publishing the result of his researches. Incidentally, it may be remarked, the objection may not be a pertinent one, because the observations scarcely cover a long enough interval of time to establish stability as an objective fact. However this may be, the question hardly arises here, because the residual series does not seem in any way suggestive of a triple system, of a simple and obvious structure at any rate. Accordingly, the only suggestion which remains is Campbell's, that we are dealing with a tidal effect, operating either in the form of a relative displacement of the star's atmosphere or of a shift in the spectral lines due to changes of pressure. From the elements of the orbit given above it follows that

$$(m+m')^{-2}m'^{3}\sin^{3}i = \cdot00228.$$

The recording spectrum is of solar (G) type, and if in the absence of any indication on the point we assume for the masses $m=m'=1$, this equation gives $i=12^{\circ}$, and hence $a=8{,}640{,}000$ km. These figures seem quite plausible, and are quite probably of the right order. It is evident that in a binary system on this scale the tide-raising forces must be sufficient to account for considerable effects.

11. If varying pressure is the controlling influence which governs the abnormal displacement of the spectral lines, this can only be detected by studying the behaviour of individual lines. Such a study would be a matter of extreme difficulty, because the shifts are very small, though their effect is plainly visible in the mean. If, on the other hand, there is an actual displacement of the star's atmosphere relative to its centre, and this is the mainly responsible cause, we should gain some idea of the magnitude of the effect by integrating $(V-V_e)$. This integral is

$$\int(V-V_e)dt=\mu^{-1}(+\cdot57\sin 3\mu t+\cdot26\sin 5\mu t-\cdot11\sin 7\mu t+\cdot06\sin 9\mu t-\cdot03\sin 11\mu t).$$

The graph of the function $(V-V_e)$ is shown in fig. 1, where the unit of the ordinates is 1 km. per sec., and the graph of the integral in fig. 2, where the unit of the ordinates is 100,000 km.,

which the extreme amplitude very slightly exceeds. The positive direction is *from* the observer. It will be noticed that fig. 1 shows that the elliptic curve crosses the interpolated curve drawn through the observations no less than ten times in a single period, but it leaves the extreme deviations as large as those given by Campbell's orbit. It seems evident enough that the velocity curve

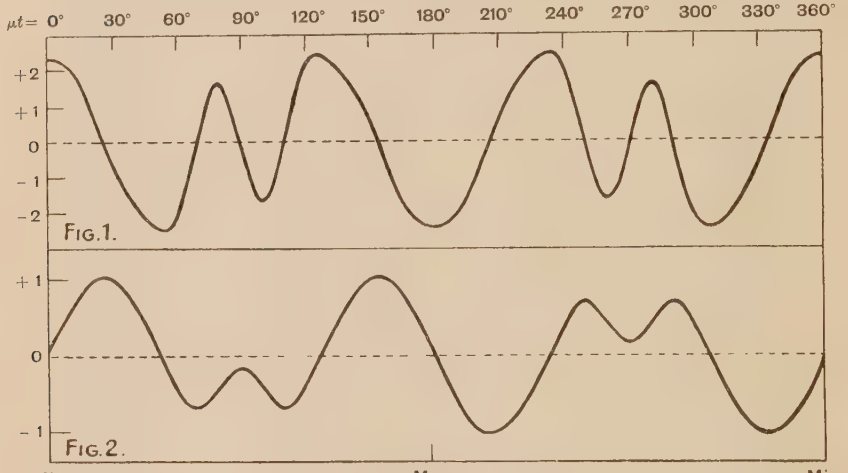


Fig. 1. $V - V_e$. Fig. 2. $\int (V - V_e) dt$.

of the star is highly complex, whatever the explanation may be, and it is difficult to believe that the light curve can really be a perfectly smooth one. We may conclude by quoting Dr. Campbell's remark: "In this connection it is very desirable that the light curve of ζ Geminorum be determined with great accuracy, to ascertain whether the irregularities in the velocity curve may not have their exact counterparts in the light curve."

Dunsink Observatory:
1913 *June* 10.

Observations of the Orionids. By W. F. Denning.

The October Orionids form one of the most interesting and important of our autumnal meteoric displays. It endures much longer than any of the other leading showers except the Perseids, and I have seen distinct traces of it during the three weeks from October 8 to 29. It affords an excellent opportunity for testing whether the radiant has, like the August Perseids, a daily motion of 1° to the eastward. In *Monthly Notices*, vol. lvi. pp. 74–9, I gave a summary of my observations of this stream, and stated that my results showed the point of radiation to be quite stationary.

Last year a paper was published by Mr. C. P. Olivier (*Transactions of the American Phil. Soc.*, xxii., part i.) in which he gave orbits for 175 meteoric streams derived from 6200 meteors recorded during the previous 10 years. These meteors were nearly all observed at the periods of the Perseids, Orionids, and Leonids. Mr. Olivier has come to certain conclusions with reference to the Orionids, and regards them as a system which has broken up into an extensive arrangement of multiple radiants. How this can have happened I know not, seeing that to me and to Schmidt, A. S. Herschel, Tupman, Corder, and many others the radiant has usually appeared beautifully defined. Of course there are some contemporary showers in the same region of the sky as are found in the case of the other rich periodical displays. But the Orionids present no very special characteristics, and seem to recur every year with the same regularity, if not with quite the same richness, as the Perseids.

Looking at Mr. Olivier's radiants of the Orionids, I am bound to consider that they strikingly corroborate the stationary aspect which is the very thing he condemns. He has adopted a method of reduction which, instead of exhibiting what the data really teach us, veils their actual meaning, and at the same time unduly complicates and multiplies the results. He has frequently broken up his observations even on the same nights, and instead of getting a good mean radiant from all the meteors, has made a lot of determinations, usually on very slender data, of what are merely fictitious outliers of one and the same radiant. Each position where a few tracks intersect near the main centre is accepted as a separate radiant, and there arise errors sometimes as great as 10°.

Thus there are seven positions closely surrounding the Orionid radiant found by Mr. Olivier at practically the same time on 1904 October 18·81 and 18·82 G.M.T., Nos. 109, 110, 111, 112, 113, 114, and 115.

These really form one shower from 89°·5, + 16°·2, and should be considered as such. Want of perfect uniformity in flight directions

always give discordances in a manner more or less pronounced. You can easily get it with the Perseids and Leonids, and it has forced itself upon my attention many times. There are several well-known reasons for these diffused radiants, viz. errors of observation, deflected meteors, and it is sometimes encouraged by meteors being allotted to their wrong radiants. But to adopt all the minor intersecting points in the radiant area of a rich shower seen in several years is simply to let scores of false "paper" radiants occupy the place of the one real radiant point existing in the sky.

To exemplify this matter I may just refer to observations of the Perseids made at the Lick Observatory in 1894 on August 9, 10, and 11 (*Meteors and Sunsets*, page 58). Now I am not going to impeach the very able astronomers who conducted and reduced those observations, because the work was simply experimental, I believe, and no great weight was attached to it. The three nights yielded 21 radiants scattered from R.A. 17° to 56°, Dec. +50° to 71°. Now the meteors forming these radiants were undoubtedly Perseids, the mean of the 21 radiants being at 40° + 58° and close to the main Perseid radiant. To retain the 21 individual points would be absurd, for they represent nothing tangible; another 21 could have been obtained in the following year and so on, with no good annual agreements, but with the assurance of an ultimate and useless jumble.

The errors of observation appear to be very large generally, in Mr. Olivier's Orionid radiants, but this is the result of employing only a few meteors in getting most of the centres. Look at Nos. 100, 114, 118, 121, 122, 124, and 125, which are based on an ample number of meteors. It is significant that these seven positions are in excellent agreement with the central point at 91°, +15°.

If we take the mean of Mr. Olivier's observations on each night, and include as Orionids all positions within 10° or so of the well-established centre of the radiant, we shall obtain a result which nearly agrees with that obtained at Bristol. This is a perfectly fair treatment of the positions. I have from my own and Mr. Olivier's observations arranged the mean places in the following table side by side. To my mind they present a satisfactory accordance, and any other treatment of the materials must, I think, seriously impair their value.

Mr. Olivier's Radiants of Orionids.

No.	Date.	Radiant.	Meteors.	Daily Means.	↓'s.	Rads.
74	1909 Oct. 13·01	91°·3 + 4°·4	3	91°·1 + 9°·2	15	4
77	13·94	87·9 + 14·6	5			
78	13·94	90·7 + 5·7	4			
79	13·94	94·4 + 12·2	3			
80	1904 14·81	95·9 + 16·1	4	95·9 + 16·1	4	1

*Mr. Olivier's Radiants of Orionids**—continued.

No.	Date.	Radiant.	Meteors.	Daily Means.	⅄'s.	Rads.
81	1905 Oct. 15·89	90°·8+20°·9	4			
92	1909 15·90	89·4+14·0	5			
93	15·90	90·6+24·0	4	92·5+18·0	22	5
94	15·90	93·6+13·3	6			
95	15·90	97·9+17·7	3			
97	1904 16·80	91·6+17·7	7	91·6+17·7	7	1
98	1903 18·83	84·2+ 8·6	4			
99	18·83	87·6+13·3	5			
100	18·83	92·1+13·6	13			
103	1901 18·81	86·8+10·8	5			
104	18·83	84·6+11·0	4			
105	18·83	91·2+14·2	5			
106	18·83	94·2+11·4	7			
109	1904 18·81	83·5+18·2	4			
110	18·81	87·5+14·4	7	89·4+14·7	101	16
111	18·82	88·9+12·9	4			
112	18·81	90·0+16·4	6			
113	18·82	90·8+16·6	5			
114	18·81	92·0+15·5	16			
115	18·81	93·4+19·4	5			
116	18·81	94·1+25·0	5			
117	1908 18·73	90·2+14·3	6			
118	1903 19·82	91·5+14·4	23			
119	19·82	92·3+11·0	4			
120	1901 19·81	82·6+11·7	5			
121	19·81	90·1+14·4	13	90·2+14·7	113	7
122	19·81	91·9+12·9	30			
123	1900 19·82	90·6+15·9	9			
124	19·82	92·1+15·0	29			
125	1905 20·88	88·7+16·2	10	93·2+17·7	15	2
126	20·88	97·8+19·2	5			
128	1909 22·92	89·4+16·6	7	90·0+17·3	15	2
129	22·92	90·5+18·1	8			
130	1905 23·86	90·3+19·3	4	90·3+19·3	4	1
133	1906 25·84	86·2+17·6	6	89·1+12·7	12	2
134	25·84	91·9+ 7·7	6			
1900–9 Oct. 13–25				Mean of 41 = 90·5+14·8	308	41

* Even should a few of the radiants which I have averaged as Orionids be distinct showers, the main result will not be materially affected.

Radiant Points of the Orionids.

Date.	Denning.* Meteors registered.	Orionids.	Radiant.	Olivier. Radiant.	Orionids.	Radiants.	Meteors registered.
Oct. 8	117	5	91° +13½°				7
9	5						
11	38	1					
12	46	6	91 +17				40
13	31	4		91°·1+ 9°·2	15	4	55
14	141	7	91 +15	95·9+16·1	4	1	23
15	223	33	92 +16½	92·5+18·0	22	5	108
16	100	16	91 +15	91·6+17·7	7	1	39
17	110	40	92 +14				
18	22	13	92 +15	89·4+14·7	101	16	331
19	52	14	90½+15½	90·2+14·7	113	7	380
20	135	49	91½+14¾	93·2+17·7	15	2	34
21	97	24	92 +14				
22	64	18	91½+15½	90·0+17·3	15	2	69
23	65	6	91 +15	90·3+19·3	4	1	28
24	43	14	91 +16				
25	261			89·1+12·7	12	2	87
26	171						49
27	42		490 +16				
28	6						
29	27	1					
Oct. 8–29	1407	257	91°·2+15°·2	91°·3+15°·7	308	41	1250

In a minor degree Mr. Olivier's Perseid observations similarly exhibit a number of erratic outlying positions, due to observational errors and other causes. He has selected and placed in a special table certain positions for the Perseid radiants (July 21–August 14), and they agree fairly well with my Ephemeris positions from night to night (*Gen. Catalogue of Radiants, Memoirs,* liii. p. 210). But Mr. Olivier should have used the more recent Ephemeris in *Monthly Notices,* vol. lxii. p. 169. In addition to the selected positions given by Mr. Olivier, there are certainly ten others in his list of 175 orbits, Nos. 28, 33, 36, 43, 47, 53, 65, 69, 70, 71, and the mean of these is 40°·0 + 55°·9, August 1–14.

Doubts have been expressed on both the reality and accuracy of radiant points based on only three or four paths, and there is serious reason for scepticism. My own belief is that only in special cases should such positions be accepted. I have sometimes adopted radiants only weakly represented by four paths, but the conditions

* Revised and extended from the table in *Monthly Notices,* vol. lvi. p. 75.

were unusual in such cases; there may have been one stationary meteor, the radiant may have been near the horizon, or the position may have agreed with that of a well-known shower. In ordinary cases I think at least five meteors are required to support a radiant. It is pointed out to me that more than one-half of Mr. Olivier's positions rest on 2, 3, or 4 meteors only, and I leave it to the judgment of astronomers as to whether it is perfectly safe to adopt such positions. The determination of radiants is a very difficult and delicate process requiring ample and accurate materials, good judgment, and experience enabling the observer to allow for every little detail in the visible aspect of the meteors. The direction of flight is the main "divining-rod," but there are others to be found in the velocity, length of path, streaks, trains, etc., of the objects. The members of a common shower exhibit family resemblances, but there are occasional exceptions.

It has been said that the results for each night should be reduced separately. I agree with this for all the richer displays. But it is neither feasible nor necessary in regard to the great majority of meteoric streams, which certainly do not furnish one meteor in five hours for one observer. The combination of several nights is desirable, so that the minor systems may be detected from a sufficient number of paths. If we confine our reductions to a single night the more feeble showers must escape recognition altogether, and we shall derive a lot of false radiants from the chance intersections of few paths. My experience is that it is not merely preferable but imperative to combine the paths gathered on several nights if we desire to get safe positions and to find the more tenuous showers.

Mr. Olivier goes over ground well surveyed a generation ago in saying that the meteors forming stationary radiants cannot move in the same orbits. No one denies this conclusion. What we need is a satisfactory explanation accounting for the projection of meteors from the same apparent points during long periods. It is far easier to find such an explanation, difficult though it may be, than to eliminate the stationary radiants from the sky. The latter I may describe, after forty years of observation, to have formed one of the most splendidly well-defined features I have ever recognised in this field of work. But it requires very sedulous, ample, and precise observation to distinguish it in a striking manner. Mr. Olivier's data are altogether insufficient. His observations are nearly all at the Perseid, Orionid, and Leonid epochs, when it is comparatively easy to collect thousands of meteor paths in a few years. If he will patiently observe with the utmost care and accuracy at least 7000 or 8000 meteors on *ordinary nights of the year*, when no rich shower is in play, and correctly determine their radiants, he will find such a large number of almost identical positions that he will have to admit they possess a special significance.

In my opinion, we need several good observers in a fine climate to dive deeply into this work, for nearly all of those who have engaged in it in past years have merely skimmed the surface. We

already have myriads of recorded paths for August and November, because the attractive prevalence of meteors at these periods has naturally induced widespread observation. There are feather-bed meteoric observers as well as "feather-bed soldiers," and very few persons are to be found out in the cold watching for meteors when they are scanty, and no rich display expected.

It is certainly a regrettable circumstance that the majority of observers, when they have acquired a fair knowledge and proficiency in this field, relinquish their labours. The recording of a few thousands of meteors forms a necessary, preliminary training for really useful work. Many of the results with which we have hitherto had to deal have been obtained by observers possessing only moderate or limited experience.

Mr. Olivier says that there is no motion in the radiant of the Leonids, but this cannot be regarded as demonstrated by his observations. They cover only the few days from November 13 to 16, and the displacement, if any, would very likely be masked by various influences. The radiant is not a point, and when we consider the errors of observation, etc., it becomes obvious that a very limited period is insufficient in which to settle any questionable feature such as that referred to. The shower certainly begins as early as November 7 and is still active on November 20, but I have not collected enough data near those dates to settle finally the question as to whether or not the radiant changes its place. But it is suggestive that the meteors I have examined in my own and other catalogues for November 7–16 and 19–20 indicate the radiant at $147^\circ + 25^\circ$ and $155^\circ + 23^\circ$ respectively. The matter must, however, be regarded as an open one pending further investigation on the basis of more ample materials.

It is certain that the radiants of 33 doubly observed Leonids, the real paths of which I computed, give no indication of a progressive motion of the radiant in R.A., but the materials are limited to four nights only, as under:—

Date.	Mean R.A. of Radiant.	No. of Meteors.
November 13	150°·7	3↓'s
14	150°·4	17
15	150°·6	10
16	150°·7	3

To the paper which I wrote on the Orionid shower in *Monthly Notices*, vol. lvi. pp. 74–9, I have found practically little to add, and from that paper there is certainly nothing I wish to withdraw. I believe that it correctly represents just what is to be seen in the sky.

Bristol:
1913 *May.*